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Application of Differential Equation to Real Life Problems for Sustainable Development in Akwa Ibom State, Nigeria

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ABSTRACT

A review on the differential equation, synergy for progress, strategies for arises, and sustainable development aimed at identifying certain aspects of a differential equation. It was discovered that differential equations can be applied in the area of radioactive decay and cooling of a body. There is no doubt therefore that a thorough grounding in the theory and application of differential equations should be a part of the scientific education of every scientist, both physical and social. Sequel to the above assertions, this study conclusively submitted that differential equations are highly essential in the area of Mathematics – Analysis, Algebra, and Topology, and therefore should be given due attention as their wide applications in the global system cannot be overemphasized.

Keywords: *differential equation, synergy, strategies for arise, sustainable development.*

INTRODUCTION

Like the late seventeenth-century work in Calculus, the earliest discoveries in differential equations were first disclosed in letters from one mathematician to another, many of which are no longer available. Perhaps since astronomy was the physical field of interest that dominated the century, most of such announcements were limited to studies in planetary motion. Today, the theory of differential equations plays a fundamental role in both pure and applied mathematics. Indeed, some of the most important ideas in Analysis, Algebra, and Topology were developed in attempts to resolve particular problems involving equations of this nature. The startling growth of computers in which large systems of differential equations are solved quickly and accurately has made formulation of scientific problems in terms of differential equations more meaningful than ever before. A differential equation is a relationship between an independent variable x, a dependent variable y, and one or more derivative of y with respect to x.

e.g. $X^2\frac{dy}{dx} - y\sin x = 0\frac{xyd^2y}{dX^2} + \frac{y\,dy}{dx} + e^{3^x} = 0$

The order of a differential equation is given by the highest derivative involved in the equation; e.g

$X^2 \frac{dy}{dx} - y^2 = 0$ 1st order ………………………………………………… (I)

$d^2y/dx^2 + 10y = \sin 2x -$ 2nd order …………………. (II)

Because, in the 2nd equation, the highest derivative involved is

$\frac{d^2y}{dx^2}$ (0^0 Neil, 1991)

FORMATION OF DIFFERENTIAL EQUATIONS

Differential equations may be formed in practice from a consideration of the physical problems to which they refer. Mathematically, they can occur when arbitrary constants are eliminated from a given function e.g.

Consider y = A sin x + B cos x, where A and B are arbitrary constants. If We differentiate, we have: $\frac{dy}{dx} = A\,cos\,x - B\,sin\,x$ and $\frac{d^2y}{dx^2} = -sin\,xcos\,x$ which is identical for the original equation, but with the sign changed

i.e. $\frac{d^2y}{dx^2} = -\mathrm{y}$ Therefore $\frac{d^2y}{dx^2} + \mathrm{y} = 0$ (this is differential equation of the 2nd order)

A function with an arbitrary constant gives a 1st order equation, while a function with two and three arbitrary constants gives 2nd and 3rd order equation respectively, so without working each out in detail, we can say that:

1. $y = e^{-2x}\ (\mathrm{A} + \mathrm{B})$ would give a 2nd order differential equation;

2. $y = \mathrm{A}\left[\frac{x-1}{x+1}\right]$ would give a 1st order differential equation (Boyce, 2001)

SOLUTION OF DIFFERENTIAL EQUATIONS

To solve a differential equation we have to find the function for which the equation is true. This means that we have to manipulate the equation to eliminate all the derivatives and leave a relationship between y and x, this is the 1st order differential equation. Every function which, when substituted, together with its derivative into the given differential equation, turns the equation into an identity is called "a solution of the differential equation".

The more general ordinary differential equation of the 1st order is an equation of the form $y = f(x, y)$ ______________ *1

Another example of ordinary differential equation of extensive practical application is the equation: $a(x)\mathrm{y}'' + b(x)y' + c(x)y = fx$______ *2, where a, b, c and f are given functions of(x). This is called linear second – order ordinary differential equation.

In equation *2 above, the right member (x), is identically zero, then the equation is referred to as "Homogenous linear equation", otherwise is called non-homogenous.

Differential equation can also be specified according to their degree in some circumstances, and it often take the form of a polynomial expression in the unknown function and its derivatives, the degree being the exponent of the highest-ordered derivatives. It is clear that all linear differential equations are of the 1st degree except $yy' + by + c = 0$ (Erwin, 1979)

Methods of Solving Differential Equation

1. **Direct Integration:** If the equation can be arrange in the form: $\frac{dy}{dx} = f\ (x)$, then the equation can be solved by simple integration.

Example: $e\frac{dy}{dx} = 4$, given that y = 3,and $x = 0$, find the particular solution?

Solution: $\frac{dy}{dx} = \frac{4}{e^x}, dy = 4e^{-x}dx, y = \int 4e^{-x}$ = -4$e^{-x} + \ c$. (gen. soln)

$3 = 4e^{-0} + \ c, y$ =-4$e^{-x} + 7$. (part. Soln)

2. **By separating the variables:** if the equation is in the form: $\frac{dy}{dx} = f(xy)$, the variable y on the right hand side (RHS) prevents solving by direct integration, and so we have to devise some other method of solution.

Let us consider equation of the form: $\frac{dy}{dx} = f(xy)$ and of the form $\frac{dy}{dx} = \frac{f(x)}{f(y)}$, that is equation which can be expressed as products or quotients of the functions of x or of y.

Example 1: solve $\frac{dy}{dx} = (1 + x)(1 + y)$

Solution: $\frac{1}{1+y}\frac{dy}{dx}dx = 1 + x$, and integrating both sides wrt x.

$\int \frac{1}{1+y}\frac{dy}{dx}dx = \int (1 + x)dx, \int \frac{1}{1+y}dy = \int (1 + x)dx$

In $(1 + y) = x + \frac{x^2}{2} + C$

Example 2: solve $\frac{dy}{dx} = \frac{y^2+xy^2}{x^2y-x^2}$

Solution: express he right hand side 1st in x-factors and y-factors

$\frac{dy}{dx} = \frac{y^2(1+x)}{x^2(y-1)}$, and now rearrange the equation so that we can have y-factors and dy, and x-factors and dx.

$\frac{y-1}{y^2}dy = \frac{(1+x)}{x^2}dx, \int \frac{y-1}{y^2}dy = \int \frac{(1+x)}{x^2}dx$
$\int \left(\frac{1}{y} - y^{-2}\right)dy = \int \left(x^{-2} + \frac{1}{x}\right)dx$, In $y + y^{-1} =$ in $x - x^{-1} + c$
In $y + y^{-1} =$ in $x - x^{-1} + c$

3. **Homogenous Equations** – By Substitution $y = vx$:
$\frac{dy}{dx} = \frac{x+3y}{2x}$ is an example of a homogenous differential equation.

This is the determination by the fact that the total degree in x and y for each of the terms involved is the same (in this case, of degree 1). The key to solving every homogenous differential equation is to substitute $y=vx$, where v is a function of x. The equation above looks simple enough, but we find that we cannot express the RHS in the form of 'x-factors' and 'y-factors'. In this case, we take $y = vx$.

Differential wrt x(Using Product Rule)
$\frac{dy}{dx} = v.1 + \frac{xdv}{dx}, v + \frac{xdv}{dx}$, Also $\frac{x+3y}{2x} = \frac{x+3vx}{2}$
The equation now becomes $v + \frac{xdv}{dx} = 1 + \frac{3v}{2} = \frac{xdv}{dx} = \frac{1+3v}{2} - v$
$1 + \frac{3v-2v}{2} = \frac{1+v}{2}$ The equation is now expressed in terms of v and x, and in the form,
We find that we can solve by separating the variables:
$\int \frac{2\,\mathrm{d}y}{1+v} = \int \frac{1\,\mathrm{d}x}{x}$ 2 In $(1 + v) =$ In $x + C =$ in $x +$ in A

$(1 + v)\underset{=}{2}Ax$. But $y = vx, \Rightarrow v = y/x$, therefore.$\left(1 + \left({}^{y}/_{x}\right)^2\right) = A = (x + y)^2 = Ax^2$
(Erwin, 1979)

LINEAR EQUATIONS – USE OF INTEGRATING FACTOR
Consider the equation $\frac{dy}{dx} + 5y = e^{2x}$
This is clearly an equation of the 1st order, but different from those we have dealt with so far.
In this case, we begin by multiplying both sides by e^{5x}, being the integrating factor, IF. This gives $e^{5x}\, dy/dx + y\, 5e^{5x}\, e^{2x}\, e^{5x} = e^{7x}$. We now find that the LHS is, in fact the derivatives of $y.e^{5x}$. Therefore, $\frac{d}{dx}(ye^{5x}) = e^{7^x}$
Integrating both sides wrt x: $ye^{5x} = \int e^{7^x}\, dx \frac{e^{7x}}{7} + C$
$y = \frac{e^{2^x}}{7} + ce^{-5x} *$ N/B: remember to divide C by e^{5x}

The equation we've just solve is an example of set of equations $\frac{dy}{dx} + py = Q$, where p and Q are functions of x (or constants).
This is called a "linear equation of the 1st order" espdx and to solve any equation, we multiply both sides by an "integrating factor" which is always. This converts the LHS into the derivative of a product (O'Neil, 1991).

In our pervious example.$\frac{dy}{dx} + 5y = e^{2x}, p = 5$
Therefore $\int p\, \mathrm{d}x = 5x$ and the integrating factor was therefore e^{5x}

NOTE: In determining$\int p\, \mathrm{d}x$, we do not include a constant of integration. This omission is purely for convenience, for a constant of integration here would in practice give a constant factor on both sides of the equation which would subsequently cancel.

EXACT DIFFERENTIAL EQUATIONS

The differential equation. $M(x,y)\, dx + N(x,y)dy = 0$…………………..*1 is called an "exact" if and only if $M(x,y)$ and $N(x,y)\frac{dN}{dy} = \frac{dN}{dx}$ and $\frac{dm}{dy} = \frac{dN}{dx}$are continuous differential functions for which the following condition is fulfilled, and are continuous in some region (Boyce, 2001)

In integrating exact differential equations, we shall prove that if the left hand side of equation *1is an exact differential, then condition* is fulfilled.
Let us first assume that the left hand side of * is an exact differential of some functions $\mathrm{U}(x,y)$, then $\mathrm{M}(x,y)dx + \mathrm{N}(x,y)dy$
$du\frac{du}{dx} + \frac{du}{dy}dy$ implies $M = \frac{du}{dx}$ and by differentiating the first relation with respect to y and the second with respect to x, we obtain
$dM = \frac{d^2\mathrm{U}}{dx\, dy}$ $\frac{d\mathrm{N}}{dx} = \frac{d^2\mathrm{U}}{dy\, dx}$, therefore condition (*) is a necessary condition for the left hand side of *1 to be an exact differential (O'Neil, 1991).
Let's see from the relation $\frac{du}{dx} = M(x,y)$ we find

$U = \int x_0 X \quad M(x,y)dx + Q(y$ __________$(*)_2$ Where x_0 is the abscissa of any point in the domain of the existence of the equation?
$\frac{dU}{dx} = \int x_0\ \frac{dM}{dy}dx + \Phi^1(y) = N(x,y)$ but $\frac{dM}{dy} =$ hence $\int{}^{x}0^{x}\ \frac{dN}{dx}\ dx$
$+\Phi^1(y) = \Phi\sqrt{(x,y)}$ i.e $N(x,y)]x_{x_0} + \Phi^1\ (y)$
$= \mathrm{N}(x,y)$or $\mathrm{N}(x,y) - \mathrm{N}(x_0 y) + \Phi^1\ (y)$

$= \mathrm{N}(x, y)$

Hence, $\Phi^1\ (y)\mathrm{N}(x_0 y)$or $\Phi(y) = \int \mathrm{N}(x_0 y)dy + \mathrm{B}$

DIFFERENTION EQUATION: SYNERGY FOR PROGRESS, STRATEGIES FOR ARISE AND SUSTAINABLE DEVELOPMENT

Differential equations provide alternative description or the real world and constitute a formidable tool convenient for modern computers. To better appreciate the role of differential equations, our attention will be restricted only across the spectrum science.

Radioactive Decay:

It is established experimentally that the rate of radioactive decay is proportional to the remaining mass of the substance. If N equals the mass of radioactive substance present at any time t, the rate of change of N is described by the equation:

$\frac{d\mathrm{N}}{dt} = -kn$, where k is proportionality constant.

The negative sign shows as decrease in the amount of the substance as t grows, and we write $\frac{d\mathrm{N}}{dt} = -kdt$, In N $= kt +$ In C or N $= Ce^{-kt}$

The value of the constant k can be determined experimentally by measuring the amount of the remaining substance at a time moment t. The rate of decay is usually characterized by the so called half-life period, T (Erwin, 1979 and James, 1999).

$\frac{N_0}{2} = N_0 e^{-kt}$ and $e^{-kt} = 2$

Radiocarbon dating is a technique used by archeologists to determine the ages of fossils uncovered during excavation (O'Neil, 1991)

Cooling of a Body

According to the law established by Newton, the temperature of a cooling body drops at a time-rate proportion to the difference between the temperature of the body and the temperature of the surrounding medium, i.e. $\frac{dT}{dT} = -k(T - Ts)$ where T is the temperature of the body (James, 1999).

Ts = Temperature of surrounding.

By separating the variables, we have: $\frac{dT}{T-Ts} = k\,dt$

In$(T - Ts)$, $= kt +$ in C, $T = Ts + C^{e^{-kt}}$

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Example for Radioactive Decay and Half-Life

Example: the n know that a certain radioactive dent in decaying at the rate proportional to the amount present. It was observed that the refined lost 30% of it original mass after 4 hours where its original mass initially was 50mg.

(i) Obtain an equation for determining the amount of material present at any time t

(ii) What is the mass after 5 hours?

(iii) What is the half-life of the radioactive element?

Solution: Let A devote the amount of material present at any time t. Then

$\frac{dA}{dt}$ x A, $\Rightarrow \frac{dA}{dt} = -KA$ ______________ (I) (- sign represent material decaying)

By separating the variables _________ (I) becomes $\frac{dA}{A} = -ktc$ and taking I_{ro} of both sides

$A = e^{-kt} + C = e^{-kt}.e^{C} \Rightarrow A = pe^{-k}$ $(p = e^{C})$ equation for the amount of material present at any time t.

(II) Mass left after 5 hours. From x, at $t = 0$, $A = 50$mg,

$\therefore$ $p = 50$

Now at 4 hours and 30% loss of its original mass $\Rightarrow \frac{30 \text{ X } 50}{100} = 15$mg

Next we find the constant K, using equation x

$\therefore 50 - 15 = 35$mg remain after 4hours

$$35 = 50e^{-k(4)} \Rightarrow 35 = 50e^{-4k}, e^{-4k} = \frac{35}{50} = 0.7$$

Taking in or both sides, $-4k = \text{In}(0.7) \Rightarrow k = \text{In}\frac{(0.7)}{4} = \frac{0.3566749}{4} = 0.089169$

$\Rightarrow A = 50e^{-(0.089169)^5} \Rightarrow A = 50e^{-0.44584}$

$\Rightarrow A = 50(0.6402830) = 32.01$mg

(III) Half-life means the time when the material decays50% of its original value.

This implies $\frac{50 \text{ X } 50}{100} = 25$mg. This is denoted by $\frac{t1}{2}$ and from x, we've:

$A = pe^{-kt} \Rightarrow 25 = 50e^{-(0.089169)^{t1/2}} \Rightarrow e^{-(0.089169)^{t1/2}} = \frac{25}{50} = 1/2,$ taking in of both sides. $-(0.089169)^{t1/2} = \text{In}(0.5)$

$$\frac{t1}{2} = \frac{\text{In}(0.5)}{0.089169} = \frac{-(0.69315)}{0.089169} \Rightarrow \frac{t1}{2} = \frac{0.69315}{0.089169} = 7.77 \approx 8 \text{ hours}$$

Example for Newton's Law of Cooling:

Example: a body with an initial temperature of 160^0c is in a room of constant temperature of 30^0c. If the temperature of the body fails to 140^0c after 20 minutes

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(i) Derive an equation for the temperature of the body at any time t.
(ii) What is the temperature of the body after 60 minutes (1hrs)?
(iii) When will the temperature fall to 350^0c ?

Solution: let temperature of the body be $\theta(t)$ and room's temperature.$\curlywedge(t)$, and by P.T.O

Newton's law of cooling, $\frac{d\theta(t)}{dt} \propto [\theta(t) - \curlywedge(t)], \frac{d\theta(t)}{dt} = -\mathrm{k}[\theta(t) - \curlywedge(t)],$

Since $\curlywedge(t) = 30^0$c

$\frac{d\theta(t)}{dt} = \mathrm{k}[\theta(t) - 3]$, and by separating variables we've:

$\frac{d\theta(t)}{[\theta(t)-30]} = -kt$, and integrating both sides,

$\mathrm{In}[\theta(t) - 30] = \int -kdt + c \Rightarrow \mathrm{In}[\theta(t) - 30] = kt + c$, and taking exponent of both sides

$\theta(t) - 30 = e^{-kt+c} = e^c \Rightarrow \mathrm{A}e^{-kt}$ (Where A $=e^{-kt}$)

$\theta(t) - 30 = \mathrm{A}e^{-kt}$ ____________________$*$ refined equation for temperature of body at any time t .

At $t = 0, \theta(t) = 160^0$c $\Rightarrow 160 - 30 = e^{-k(0)}$

$130 = \mathrm{A}e^0 \Rightarrow \mathrm{A} = 130.$

At $t = 20, \theta(t) = 140^0$c, therefore *: $140 - 30 = 130e^{-k(20)}$

$110 = 130e^{-20k}, e^{-20k}\frac{110}{130} = 0.08462$

Taking in of both sides, $-20k = \mathrm{In}(0.08462), \mathrm{k} = \mathrm{In}\frac{(0.08462)}{20} = -\frac{(0.1669)}{20}$

$$\mathrm{k} = \frac{(0.1669)}{20} = 0.008349$$

(ii) When $t = 60$minutes (1hr)$, \theta(t) =$? add from *

$\theta(t) - 30 = \mathrm{A}e^{-kt} \Rightarrow \theta(t) - 30 = 30e^{-(0.008345)(60)} = 130e^{-0.50094}$

$\theta(t) - 30 = 130(0.605961) \Rightarrow 78.76493$

$\theta(t) - 78.76493 + 30 = 108.77$

$\theta(t) \cong 109^0$c

(iii) When $\theta(t) = 35^0$c$, = t =$? and from *, $\theta(t) = 35^0$c, we've:

$35 - 30 = 130e^{(0.008349)t} \Rightarrow 5 = 130e^{-(0.008349)t}$

$e^{-(0.008349)t} = \frac{5}{130} = 0.0385$, and taking in of both sides,

$-0.08349t = \mathrm{In}(0.0385)$

$t = \frac{\mathrm{In}(0.0385)}{0.008349} = \frac{-3.2571}{-0.08349} = 390$minutes

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CONCULDING REMARKS

The aim of this study was to identify certain aspects of environmental related phenomena which can be strategized for arise and sustainable development using differential equations. The study highlighted that differential equations can be applied in the area of radioactive decay, cooling of a body, Hence, for the synergy, strategies for arise and sustainable developments to work well in the area of Mathematics, the core area of Mathematics (pure and applied),the differential equations with its wide applications should be given priority if the entire science is to be enhanced experimentally through manpower training and adequate workshops organized by the aboard-trained Mathematicians and Engineers. Differential equation is highly essential in the area of Mathematics – Analysis, Algebra and Topology, and therefore should be given due attention as its wide applications in the global system cannot be overemphasized.

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