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Application of Partial Derivatives to Real Life Issue and Sustainable Development in Akwa Ibom State and Nigeria

Umoh, Ezekiel S.
General Studies Department
Akwa Ibom State Polytechnic, Ikot Osurua, Ikot Ekpene, Nigeria
E-mail: elzekumoh@gmail.com

ABSTRACT

The theoretical study of curves and surfaces began more than two thousand years ago when the Greek philosopher-mathematician explored the properties of conic sections, helixes, spirals, and surfaces of revolution generated from them. While applications were not on their minds, many practical consequences evolved these included the representation of the elliptical paths of the planet about the sun, the employment of the focal properties of the paraboloids, and the use of the special properties of helixes to construct the double-helical model of DNA (Deoxyribonucleic acid). The analytic tool for studying functions of more than one variable is the partial derivative. Surfaces are a geometric starting point since they are presented by functions of two independent variables; and in this context, the coordinate equations will be exhibited.

Keywords: partial derivatives, synergy, strategies for arise, sustainable development.

INTRODUCTION

Let Z = $f(x, y)$ be a function of two variables. If x varies while y is held fixed, Z becomes a function of x.then its derivative for lim $\Delta x \to 0$ $\frac{f(x+\Delta xy)-f(xy)}{\Delta x}$ is called the (1st) partial derivative to x, and is denoted by $\frac{\partial f}{\partial x}$ or $\frac{\partial z}{\partial x}$ fx (x, y). Similarly, if y varies while x is held fixed the (1st) partial derivative of f concerning y is $\frac{\partial f}{\partial x}$ or $\frac{\partial z}{\partial x}$ or f_y (x, y) = lim $\Delta x \to 0$ $\frac{f(x+\Delta xy)-f(xy)}{\Delta x}$ (Ayres, 1990).

Definition of Partial Derivatives

The ordinary derivative of a function of several with respect to one of the independent variables, keeping all other independent variable constant, is called the partial derivative

of function with respect to the variables. Partial derivatives of f (x, y) with respect to x and y are denoted by
$\frac{\partial f}{\partial x}\left[fx'fx(x,y),\frac{\partial f}{\partial x}\mid y\right]$ and $\frac{\partial f}{\partial x}\left[or\ fy'fy(x,y),\frac{\partial f}{\partial y}\mid x\right]$ respectively, the latter notation being used when needed to emphasize which variables are held constant.

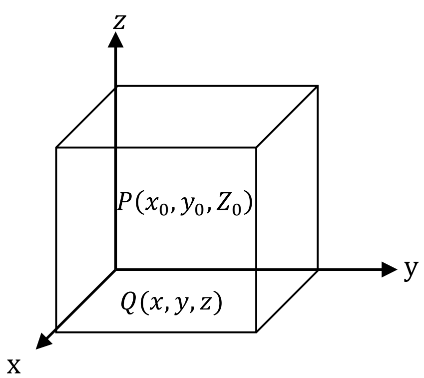
By definition, $\frac{\partial f}{\partial x}$ $\lim \Delta x \to 0$ $\frac{f(x+\Delta xy)-f(xy)}{\Delta x}$,
$\frac{\partial f}{\partial y} = \lim \Delta y \to 0$ $\frac{f(x+\Delta xy)-f(x,y)}{\Delta y}$ ---------- (1)

When these limits exist. The derivative evaluated at the particular point $(x_0, y_0,)$ are often indicated by:
$\frac{\partial f}{\partial x}$ $|(x_0, y_0,) = f_x\ (x_0, y_0,)$ *and* $\frac{\partial f}{\partial y}|\ (x_0, y_0,) = f_y\ (x_0, y_0,)$ *respectively*
E.g. if $f(x, y) = 2x^3+3xy^2$, then $f_x = \frac{\partial f}{\partial x} = 6x^2+3y^2$ and
$f_y = \frac{\partial f}{\partial y} = 6xy$. Also $f_x(1,2) = 6(1)^2 + 3(2)^2 = 18$, and $f_y(1,2) = 6(1)(2) = 12$ (Wrede, 2002).

If a function f has continuous partial derivative of is $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ in a region, then f must be continuous in the region. However, the existence of these partial derivatives alone is not enough to guarantee the continuity of the function and higher order partial derivatives shall not be discussed here (Spiegel, 2010).

Some Practical Applications of Partial Derivatives on Some Surfaces and Curves

A. Tangent Plane to a Surface
Let $f(x, y, z) = 0$ be the equation of a surface S such as shown below



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Assume that F, and all other functions are continuously differentiable unless otherwise indicated, and we wish to find the equation of the tangent plane to S at the point $P(x_0, y_0 Z_0)$. A vector normal to S at this point will be $N_0 \nabla Fp$ where p is the gradient to be evaluated at point $P(x_0, y_0 Z_0)$. If r_0 and r are tangents, the vectors drawn, respectively, from 0 to $P(x_0,\ y_{0,}\ Z_0)$ and $Q(x, y, z)$ on the tangent plane, the equation of the plane is (r - r_0). $N_0 = r - r_0 . \nabla Fr = 0$, since $r - r_0$ is perpendicular to N_0. In rectangular form this is:

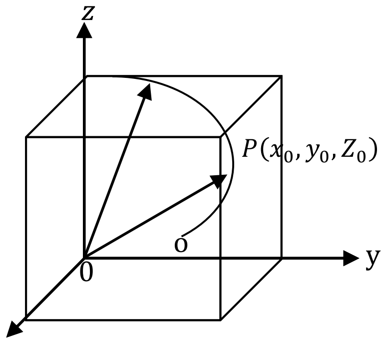
$$\frac{\partial f}{\partial x}\ p(x_0 x_0) + \frac{\partial f}{\partial y}\bigg|_P (y - y_0) + \frac{\partial f}{\partial z}\bigg|_P (z - z_0)$$ (Werede, 2010)

B. Normal Line Surface

If we now let r be the vector drawn from origin, 0 to any point (x, y, z) on the normalN_0, we see that (r - r_0) is collinear with N_0 and so, the required equation is that (r - r_0) x N_0= (r - r_0) x$\nabla F/r$=0 and by expressing the cross product in the determination form:

$$\begin{vmatrix} I & j & k \\ x - x_0 & y - y_0 & z - z_0 \\ Fr/p & Fr/p & Fr/p \end{vmatrix}$$ We find that $\frac{(x - x_0)}{\frac{\partial f}{\partial x}|r}\quad \frac{(y - y_0)}{\frac{\partial f}{\partial x}|r}\quad \frac{(z - z_0)}{\frac{\partial f}{\partial x}|r}$ (Spiegel, 2002).

C. Tangent Line to A Curve



Let the parametric equation of curve C be *x= f(u), y=g(u) and z=h(u)*, where f, g and h are continuously differentiable. If *R = f (u) i + g(u) j + h(u)k*, then a vector tangent to C at the point P is given thus: $T_0 = \frac{\partial R}{du}|r$. In rectangular form this becomes

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$$\frac{x-x_0}{\begin{vmatrix} Fy & Fz \\ Gy & Gz \end{vmatrix}} = \frac{y-y_0}{\begin{vmatrix} Fy & Fz \\ Gy & Gz \end{vmatrix}} = \frac{z-z_0}{\begin{vmatrix} Fy & Fz \\ Gy & Gz \end{vmatrix}}$$

D. Normal Plane to a Curve

If we want to find an equation for the normal plane to curve C at $P\ (x_0, y_0 Z_0)$ in the figure above, and letting r be the vector from 0 to any point (*x, y, z*) on the plane it follows that r - r_0 is perpendicular to T_0 and the required equation id the intersect of the implicitly defined surfaces *F*(*x, y, z*) = 0 and *G*(*x, y, z*) = 0,

Then
(Spiegel, 2010).

Partial Derivatives of Higher Order

We take the partial derivatives with respect to *x* and *y* of $\frac{\partial f}{\partial x}$, yielding
$\frac{\partial^2 z}{\partial x^2} = f_{xx}\ (x, y,) = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$ and $\frac{\partial^2 z}{\partial x \partial y} = f_{yx}\ (x, y,) = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$.

Similarly, from $\frac{\partial z}{\partial y}$, we obtain
$\frac{\partial^2 z}{\partial x^2} = f_{yy}\ (x, y,) = \frac{\partial}{\partial y}\left(\frac{\partial x}{\partial y}\right)$ and $\frac{\partial^2 z}{\partial x \partial y} = f\ (x, y,) = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$ (Ayres, F. 2009)

Evaluation of Partial Derivatives of Functions

Evaluate the 1st partial derivatives of Z with respect to the independent variables *x* and *y* in the following functions:

a) $x^2 + y^2 + z^2 = 25$ b) $x^2(+2y + 3z) + y^2(3x - 4z) + z^2(x - 2y) = xyz$

Solution

a) Differentiating it implicitly and treating *y* as constant, $2x + 2z\frac{\partial z}{\partial x} = 0$, hence

$$\frac{\partial z}{\partial x} = {-x}/{z}$$

Differentiating implicitly with respect to *y*, treating *x* as a constant

Hence $\frac{\partial z}{\partial y} = {-y}/{z}$

b) Differentiating implicitly with respect to *x:*

Solving for $\frac{\partial z}{\partial x}$ yields:

Differentiating implicitly with respect to y

$$2x^2 + 3x^2\frac{\partial z}{\partial y} + 2y(3x - 4z) - 4y^2\frac{\partial z}{\partial y} + 2z(x - 2y)\frac{\partial z}{\partial y} - 2z^2 = xz + xy\frac{\partial z}{\partial y}$$

And solving for $\frac{\partial z}{\partial y}$ yields:

$$\frac{\partial z}{\partial y} = \frac{-2x^2+6xy-8yz-2z^2-xz}{3x^2-4y^2+2xz-4yz-xy}$$ (Mendelson, 1999)

E. **Envelopes:** solution of the differential equations in two variables is geometrically represented by one-parameter families of curves. Some such a family characterizes a curve called an "Envelop". For instance the family of all lines, one unit from the origin may be represented by x sin $a - y$cos a-1 = 0, where a is a parameter. The 'Envelop' of this family is the circle $x^2 + y^2 = 1$. If $\Phi(x, y, z)$ = 0, is a one parametric family of curves in the xy-plane, there may be a curve E which tangent at each point to some members of the family, and such that each member of the family is tangent to E. if E exists, its equation can be found by solving simultaneously the equation, $\Phi(x, y, a)$ = 0, and E is called 'Envelop of the family", $\Phi a(xya)$ = 0.

F. Error and Application Calculation: Let $Z = f(x, y)$ be function of two independent variable x and y. Suppose those are changes in x and y, by Δx and Δy respectively. Then the function $f(x, y)$ becomes $f(x + \Delta x, y + \Delta y)$. Suppose further that there is also change in Z, then we've: $z + \Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$, and subtracting Z from both sides, we've: $\Delta z = f(x + \Delta x, y + \Delta y)$ on the RHS,

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y + \Delta y) + f(x, y + \Delta y) - f(x, y)$$

$$\Delta z = \frac{f(x + \Delta x, y + \Delta y) - f(x, y + \Delta y)}{\Delta x}\Delta x + \frac{f(x + \Delta xy + \Delta y) - f(x, y)\Delta y}{\Delta y}$$

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$\Delta z = \frac{\partial z}{\partial x}\,\partial x + \frac{\partial z}{\partial y}\,\Delta y$ Higher powers which are of very small qualities that can be rejected. Hence, $\Delta z = \frac{\partial z}{\partial x}\,\partial x + \frac{\partial z}{\partial y}\,\Delta y$ (Amos, 2015).

The quality Δx in the absolute error and $\frac{\Delta x}{x}$ in the relative error while $\left(\frac{\Delta x}{x}\right)$ x 100 in the percentage error of the function or quantity.

Example (1): The dimensions of a cone are: radium 4m, length 6m. What in the error in its volume if the scale used in taking the measurement in short by 0.0tm per meter?

Solution: volume V of a are in given by, V = $1/_3\,\pi r^2 h$ ______________ (I)

$\frac{\partial v}{\partial r} = 2/_3\,\pi r h, \frac{\partial V}{\partial h} = \frac{\pi r^2}{3}$

Since the measuring instrument is short by 0.0tm per meter, then

$\Delta r = (0.01)4 = 0.04, \quad \Delta h = (0.01)6 = 0.06$

$\Delta v = \frac{\partial v}{\partial r}\Delta r + \frac{\partial v}{\partial h} \Rightarrow 2/_3\,\pi r h(0.04) + \frac{\pi r^2}{3}\,(0.06)$______________(II)

But $\mathrm{r} = 4\mathrm{m}, \mathrm{l} = 6\mathrm{m}$, hence (II) $\Rightarrow \Delta \mathrm{v} = 2/_3\,\pi(4)(6)(0.04) + \frac{\pi(4)^2(0.06)}{3}$

$\Rightarrow 0.64\pi + 0.32\pi \quad \Rightarrow \Delta v = 0.96\pi m^3$

Example (II): The power dissipated in a retritor in given by $\mathrm{P} = \frac{E^2}{R}$. If $\mathrm{E} = 200\mathrm{volts}$ and $\mathrm{R} = 8$ ohms. Find the approximate change in P resulting from a drop of 5volts in E and an increase of 0.2ohm in R.

Solution: $\mathrm{P} = \frac{E^2}{R}, \frac{\partial \mathrm{p}}{\partial E} = \mathrm{R}\frac{(2\mathrm{E}) - E^2(0)}{R^2} = \frac{2\mathrm{ER}}{R^2} = \frac{\text{"2E"}}{R^2}$

$\frac{\partial \mathrm{p}}{\partial E} = \frac{\mathrm{R}(0) - (E^2)(1)}{R^2} = \frac{-E^2}{R^2} \quad = \Delta\mathrm{E} = -5$ (negative sign since volts is dropping),

$\Delta\mathrm{R} = 0.02$

$\mathrm{E} = 200\mathrm{volts}, \mathrm{R} = 8$ ohms.

$\therefore\ \Delta\mathrm{p} = \frac{\partial \mathrm{p}}{\partial E}\Delta\mathrm{E} + \frac{\partial \mathrm{p}}{\partial R}\,\Delta\mathrm{R} = \left(\frac{2E}{R}\right)(-5) + \left(\frac{-E^2}{R^2}\right)(0.2)$, substituting values of E & R we've:

$\Delta\mathrm{p} = 2\frac{(200)(-5)}{8} - \frac{(200)^2(0.2)}{64} \ \Rightarrow\ \Delta\mathrm{p} = -250 - 125 = -375$

Hence, P is decreased by 375watt.

CONCLUDING REMARKS

The aim of this study was to outline certain aspects of partial derivatives as they relate to the surface – curves, helixes, among others on the renewable energy and its sustainability. The study highlighted that partial derivatives can be applied on the surfaces of revolution, tangents, plane surfaces and curves which have wider applications on geophysical phenomena. Hence, the strategic for arise and partial derivatives of functions must be given priority if the entire science is to be enhanced experimentally through manpower training and adequate workshops organized. Partial derivatives is highly essential in the area of mathematics – analysis, algebra, trigonometry and topology, and should be given due attention and every person in all fields of endeavor must be equipped with the knowledge of these surfaces or shape and also be partially science-inclined for global revolution.

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