Journal of Research in Education and Society, Volume 15, Number 1, April 2024
ISSN(p): 2141-6753 ISSN(e): 2795-3033
Published By
International Centre for Integrated Development Research, Nigeria
In collaboration with
Copperstone University, Luanshya, Zambia

A Method for the Solution of Quadratic Equations Based On Difference of Squares

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ABSTRACT

The paper reviews a procedure for the general solution of certain types of quadratic equations. Earlier Eno D. John (2017) showed of a quadratic equation can be obtained through a process that reduces the equation into a difference of two squares for $a > 1$. *Under this review, we studied the implementation of the procedure when* $a < 0$. *Examples are given at the end of this work to demonstrate the method.*

Keywords: *Discriminant, perfect squares, general solution, difference of two squares*

1.0 INTRODUCTION

The equation

$$at^2 + bt + c = 0, a \neq 0 \qquad (1)$$

known as quadratic equation under certain conditions.
is commonly associated with models of parabolic functions, Parent (2015), Aravind Narayan (2013), with the roots defined by the intersection of the curve with the t- axis of the function $y = at^2 + bt + c$. Aside from quadratic equations being studied as a topic in mathematics, other areas of research in sciences considers it as a method of investigation of scientific experiments, , Parker (1977), Sergey (2006)

Some of the methods employed for the solution of (1) can be seen in Stroud & Booth (2001), Nayak & Dash (2013), Rich and Schmidt (2004) which are:

(i) Factorization
(ii) Completing the squares

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(iii) General formula
(iv) Graphical method

The general solution (iii) above of (1) is a solution of the form

$$t = \frac{-b \pm \sqrt{\Delta}}{2a} \qquad (2)$$

where Δ is the discriminant defined by

$$\Delta = b^2 - 4ac \qquad (3)$$

From (1) and (3)
if
$\Delta < 0$, equation (1) has complex roots
$\Delta > 0$, equation (1) has real and distinct roots
$\Delta = 0$, equation (1) has real and equal roots
Method (i) is employed when Δ is a perfect square and (ii) if Δ is not a perfect square. Equation (1) was studied under the assumption that $a > 1$, and a procedure developed for obtaining (2) using the method of difference of squares, Eno D. John (2017). It was shown that for any quadratic equation satisfying prescribed conditions, the method described here can be used to obtain its roots.

In this paper, we consider (1) under the assumption $a < 0$.

In the following section, we shall give the propositions necessary for the construction of the solution of (1) with $a < 0$. In section 3, the method developed in this paper shall be demonstrated with examples to illustrate the applicability of the method.

2.0 Basic Propositions for the Method of Difference of Squares

Proposition 2.1
Let $\Delta \geq 0$ for equation (1), if in addition $a = -b^2$, then equation (1) will have a real root if and only if $c > 0$.

Proof:
Let $\Delta = b^2 - 4ac$, from (3),
then for $a = -b^2$

$$\Delta = b^2 - 4(-b^2)c$$
$$= b^2(1 + 4c).$$

$\Delta \geq 0$ and will remain positive if and only if $c > 0$ with (1) having real roots.

Proposition 2.2
Let a be a perfect square, if $\sqrt{a}$ is a factor of b then the expression $bt - at^2$ is factorizable as a difference of two squares.

Proof:
Let $a = k^2$and $b = kn$ for some integers k, n
then

$$bt - at^2$$
$$= knt - k^2t^2$$
$$= knt - (kt)^2$$
$$= kt(n - kt)$$
$$= \left[\frac{n}{2} - \left(\frac{n}{2} - kt\right) - \right]\left[\frac{n}{2} + \left(\frac{n}{2} - kt\right)\right] \quad (4)$$

Choosing

$$\frac{n}{2} = X \text{ and } Y = \frac{n}{2} - kt$$

Equation (4) becomes

$$(X - Y)(X + Y)$$
$$= X^2 - Y^2.$$

Proposition 2.3
Given equation (1) with $c = 0$ and $a < 0$, then (1) can be expressed as difference of two squares.

Proof:
For $c = 0$, (1) becomes

$$bt + (-a)t^2 = 0$$

Multiplying both sides of the equation by a

$$abt - (at)^2 = 0$$
$$at(b - at) = 0$$
$$\left[\frac{b}{2} - \left(\frac{b}{2} - at\right) - \frac{b}{2}\right]\left[\frac{b}{2} + \left(\frac{b}{2} - at\right)\right] = 0$$

by Proposition 2.2.
This means

$$bt - at^2 = 0$$

is equivalent to

$$X^2 - Y^2 = 0$$

with

$$X = \frac{b}{2} \text{ and } Y = \frac{b}{2} - at$$

3.0 Implementing Method of Difference of Squares for Quadratic Equations

From equation (1), let $a < 0$, then

$$c + bt - at^2 = 0$$

and

$$bt - at^2 = -c.$$

Multiplying both sides by a,

$$abt - (at)^2 = -ac$$
$$at(b - at) = -ac$$
$$\left[\frac{b}{2} - \left(\frac{b}{2} - at\right)\right]\left[\frac{b}{2} + \left(\frac{b}{2} - at\right)\right] = -ac$$
$$\left[\frac{b}{2}\right]^2 - \left[at + \frac{b}{2}\right]^2 = -ac$$

by proposition 2.2 and 2.3.

$$\left[\frac{b}{2} - at\right]^2 = \frac{b^2}{4} + ac = \frac{b^2 + 4ac}{4}$$
$$at - \frac{b}{2} = \mp\frac{\sqrt{b^2 + 4ac}}{2}$$
$$at = \frac{b}{2} \mp \frac{\sqrt{b^2 + 4ac}}{2}$$
$$t = \frac{b \mp \sqrt{b^2 + 4ac}}{2a} \qquad (5)$$

For $a = -k$, for any number k,

$$t = \frac{-b \pm \sqrt{b^2 - 4kc}}{2k} = t = \frac{b \mp \sqrt{\Delta}}{2a} \qquad (6)$$

Example 3.1
Determine the types of roots in the quadratic equation given by $3 + 2t - 4t^2 = 0$.

Solution
$a = b^2$ and since $c = 3 > 0$
This equation has real roots by proposition 2.1.

Example 3.2
Solve the equation $-4t^2 + 12t + 8 = 0$ using the method of difference of squares

Solution
From $-4t^2 + 12t + 8 = 0$
$|a|$ is a perfect square and $\sqrt{|a|}$ is a factor of b
Thus, by propositions 2.2 and 2.3

$$-4t^2 + 12t = -8$$
$$X = 3, Y = 3 - 2t.$$

Thus,

$$3^2 - [3 - 2t]^2 = -8$$
$$2t - 3 = \pm\sqrt{17}$$
$$t = \frac{3 \pm \sqrt{17}}{2}$$

Example 3.3
Solve the equation $-3t^2 + 5t + 4 = 0$ using the method of difference of squares

Solution
From $-3t^2 + 5t + 4 = 0$
$|a|$ is not a perfect square and $\sqrt{|a|}$ is not a factor of b.

$$-3t^2 - 5t = -4$$

Multiplying both sides by 3 (coefficient of t^2)

$$-9t^2 + 15t = -12$$
$$3t(5 - 3t) = -12$$

hence,

$$X = \frac{5}{2}, Y = \frac{5}{2} - 3t.$$

So

$$X^2 - Y^2 = \left[\frac{5}{2}\right]^2 - \left[\frac{5}{2} - 3t\right]^2$$

and

$$\left[\frac{5}{2}\right]^2 - \left[\frac{5}{2} - 3t\right]^2 = -12$$
$$\left[\frac{5}{2} - 3t\right]^2 = \frac{73}{4}$$

$$3t = \frac{5 \pm \sqrt{73}}{2}$$
$$t = \frac{5 \pm \sqrt{73}}{6}.$$

4.0 CONCLUSION

The review of the earlier work was based on certain assumptions on a, noting that such assumptions are satisfied for a lot of quadratic equations as shown in the examples above. Any method chosen for the solution of quadratic equations must be subject to certain conditions on a, b and c. The examples given in this work successfully established the earlier propositions stated in section 2. The extension of this work to higher polynomial equations is currently under consideration.

REFERENCES

Aravind Narayan, (2013), Graphing the Complex Roots of Quadratic Functions on a Three-Dimensional Coordinate Space, IOSR Journal of Mathematics, 5(5), 27-36

Eno D. John, (2017), An Alternative Method for the Solution of Quadratic Equations, Akwapoly journal of Communication and Scientific Research, 2(1) 2016

Makbule Gözde Didiş Kabar, (2023), A Thematic Review of Quadratic Equation Studies in The Field of Mathematics Education, Participatory Educational Research (PER) Vol.10(4), pp. 29-48

Nayak T. and Dash, T. (2013). Solution of Quadratic Equations using Genetic Algorithm, Proceedings of national Conference on AIRES. http:arxiv.org/ftp/arxiv/papers/1306

Parent J. S. S. (Students' Understanding of the Quadratic function: Learning from Students' Voices, Dissertation, University of Vermont, schorlarlyworks.uvm.edu

Parker, G. W. (1977). Projectile Motion with Air resistance Quadratic in Speed, American Journal of Physics, 45(7), 606 – 610.

Sergey, G. (2006). The Maynard Smith Model of Sympatric Speciation, Journal of Theoretical Biology, 239, 172 – 182.

Rich, B. & Schimdt, P. (2004). Schaum Outline of Theory and Problems of Elementary Algebra, Magraw Hill Company.

Stroud, K. A. and Booth, D., (2001), Engineering Mathematics, Fifth Edition, Palgrave New York.