Some Applications of Partial Derivatives on Renewable Energy for Sustainable Development

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ABSTRACT

The theoretical study of curves and surfaces began more than two thousand years ago when the Greek philosopher - mathematician explored the properties of conic sections, helixes, spirals and surfaces of revolution generated from them. While applications were not on their minds, many practical consequences evolved. These included representation of the elliptical paths of planets about the sun, employment of the focal properties of paraboloids and use of the special properties of helixes to construct the double helical model of DNA (Deoxyribonucleic acid). The analytic tool for studying functions of more than one variable is the partial derivative. Surfaces are a geometric starting point, since they are represented by functions of two independent variables; and in this context, the coordinate equations will be exhibited.

Keywords: *Partial derivatives, renewable energy, applications, sustainable development*

INTRODUCTION

Let $Z = f(x, y)$ be a function of two variables. If x varies while y is held fixed, Z becomes a function of x. Then its derivative with respect to x

is called the (1st) partial derivative of f with respect to x, and is denoted by $\frac{\partial f}{\partial x}$ *or* $\frac{\partial Z}{\partial x}$ *or* $f_x(x, y)$. Similarly, if y varies while x is held fixed, the (1st) partial derivative of f with respect to y is

= (Ayres, 1990).

DEFINITION OF PARTIAL DERIVATIVES

The ordinary derivative of a function of several variables with respect to one of the independent variables, keeping all other independent variables constant, is called the partial derivative of the function with respect to the variable. Partial derivative of $f(x, y)$ with respect to x and y are denoted by

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$\frac{\partial f}{\partial x}\left[or \; f_x, \; f_x(x,y), \frac{\partial f}{\partial x}\Big|_y \right]$ and $\frac{\partial f}{\partial y}\left[or \; f_y, \; f_y(x,y), \frac{\partial f}{\partial y}\Big|_x \right]$, respectively, the latter notations being used when needed to emphasize which variables are held constant.

By definition, $\frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$ ------------- (1),

$$\frac{\partial f}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$

when these limits exist. The derivatives evaluated at the particular point (x_0,y_0) are often indicated by; $\frac{\partial f}{\partial x}\Big|_{(x_0,y_0)} = f_x(x_0,y_0)$ and $\frac{\partial f}{\partial y}\Big|_{(x_0,y_0)} = f_y(x_0,y_0)$ respectively,

e.g. If $f_y = \frac{df}{dy} = 6xy.$

Also $f_x(1,2) = 6(1)^2 + 3(2)^2 = 18$, and $f_y(1,2) = 6(1)(2) = 12$. (Wrede, 2002).

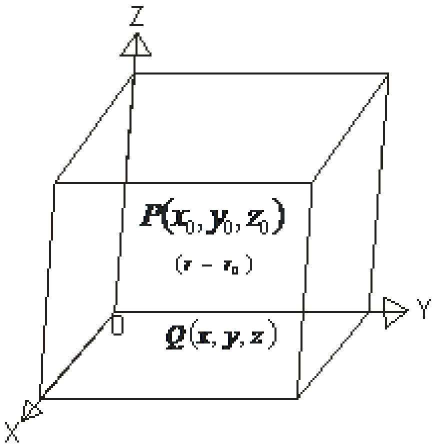
If a function f has continuous partial derivative of $\frac{df}{dx}, \frac{df}{dy}$ in a region, then f must be continuous in the region. However, the existence of these partial derivatives alone is not enough to guarantee the continuity of the function and higher order partial derivatives shall not be discussed here (Spiegel, 2010).

$f(x,y) = 2x^3 + 3xy^2$, then $f_x = \frac{\partial f}{\partial x} = 6x^2 + 3y^2$ and

Some Practical Applications of Partial Derivatives on Some Surfaces and Curves

A. *Tangent Plane to a Surface*

Let $F(x,y,z)=0$ be the equation of a surface S such as shown below.



Assume that F, and all other functions are continuously differentiable unless otherwise indicated, and we wish to find the equation of the tangent plane to S at the point $P(x_0, y_0, z_0)$. A vector normal to S at this point will be $N_0 = \nabla F\big|_p$ where p is the gradient to be evaluated at point $P(x_0, y_0, z_0)$. If r_o and r are tangents, the vectors drawn, respectively, from 0 to $P(x_0, y_0, z_0)$ and $Q(x, y, z)$ on the tangent plane, the equation of the plane is $(r - r_0).\ N_0 = (r - r_0).\nabla F\big|_r = 0$, since $r\text{-}r_0$ is perpendicular to N_0. In rectangular form this is:

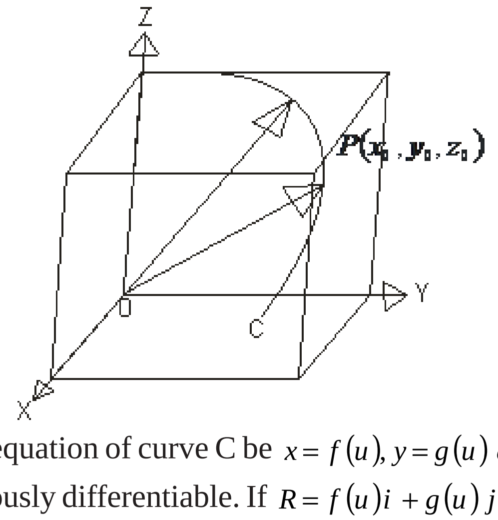
$$\frac{\partial F}{\partial x}\Big|_p (x - x_0) + \frac{\partial F}{\partial y}\Big|_p (y - y_0) + \frac{\partial F}{\partial Z}\Big|_p (z - z_0) = 0$$ (Wrede, 2010).

B. *Normal Line to a Surface*

If we now let r be the vector drawn from origin, 0 to any point (x, y, z) on the normal N_0, we see that $(r - r_0)$ is collinear with N_o and so, the required equation is $(r - r_0) \times N_0 = (r - r_0) \times \nabla F / r = 0$ and by expressing the cross product in the determinant form:

$$\begin{vmatrix} i & j & k \\ x - x_0 & y - y_0 & z - z_0 \\ F_x/p & F_y/p & F_z/p \end{vmatrix},$$ we find that $$\frac{(x - x_0)}{\frac{\partial F}{\partial x}\Big|_p} = \frac{y - y_0}{\frac{\partial F}{\partial y}\Big|_p} = \frac{z - z_0}{\frac{\partial F}{\partial z}\Big|_p}$$ (Spiegel, 2002).

C. *Tangent Line to A Curve*



Let the parametric equation of curve C be $x = f(u), y = g(u)$ *and* $z = h(u)$, where f, g and h are continuously differentiable. If $R = f(u)i + g(u)j + h(u)k$, then a vector tangent to C at the point P is given thus: $T_0 = \frac{dR}{du}\Big|_p$. In rectangular form this

becomes $$\frac{x - x_0}{\begin{vmatrix} Fy & Fz \\ Gy & Gz \end{vmatrix}_p} = \frac{y - y_0}{\begin{vmatrix} Fz & Fx \\ Gz & Gx \end{vmatrix}_p} = \frac{z - z_0}{\begin{vmatrix} Fx & Fy \\ Gx & Gy \end{vmatrix}_p}$$ (Wrede, 1963).

D. *Normal Plane to a Curve*

If we want to find an equation for the normal plane to curve C at $P(x_0, y_0, z_0)$ in the figure above, and letting r be the vector from 0 to any point (x, y, z) on that plane, it follows that $r - r_0$ is perpendicular to T_0 and the required equation is the intersect of the implicitly defined surfaces $F(x, y, z) = 0$ and $G(x, y, z) = 0$,

then

(Spiegel, 2010).

Partial Derivatives of Higher Order

We take the partial derivatives with respect to x and y of $\frac{df}{dx}$, yielding

$$\frac{\partial^2 z}{\partial x^2} = f_{xx}(x, y) = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \text{ and } \frac{\partial^2 z}{\partial x \partial y} = f_{yx}(x, y) = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right).$$

Similarly, from $\frac{\partial Z}{\partial y}$, we obtain

(Ayres, F. 2009).

$$\frac{\partial^2 Z}{\partial y^2} = f_{yy}(x, y) = \frac{\partial}{\partial y}\left(\frac{\partial Z}{\partial y}\right) \text{ and } \frac{\partial^2 Z}{\partial x \partial y} = f_{xy}(x, y) = \frac{\partial}{\partial x}\left(\frac{\partial Z}{\partial y}\right)$$

$$\begin{vmatrix} F_y & F_z \\ G_y & G_z \end{vmatrix}_p (x - x_0) + \begin{vmatrix} F_z & F_x \\ G_z & G_x \end{vmatrix}_p (y - y_0) + \begin{vmatrix} F_x & F_y \\ G_x & G_y \end{vmatrix}_p (z - z_0) = 0$$

$$2y + 2z\frac{\partial z}{\partial y} = 0$$

Evaluation of Partial Derivatives of Functions

Evaluate the 1st partial derivatives of Z with respect to the independent variables x and y in the following functions:

a) $x^2 + y^2 + z^2 = 25$ b) $x^2(+2y + 3z) + y^2(3x - 4z) + Z^2(x - 2y) = xyz$

Solution

a) Differentiating it implicitly and treating y as a constant, $2x + 2z\frac{\partial z}{\partial x} = 0$, hence

$$\frac{\partial z}{\partial x} = -x/z$$

Differentiating implicitly with respect to y, treating x as a constant ,

hence $\frac{\partial z}{\partial y} = -y/z$

b) Differentiating implicitly with respect to x:

Solving for $\frac{\partial z}{\partial x}$ yields:

Differentiating implicitly with respect to y

$$2x^2 + 3x^2 \frac{\partial z}{\partial y} + 2y(3x-4z) - 4y^2 \frac{\partial z}{\partial y} + 2z(x-2y)\frac{\partial z}{\partial y} - 2z^2 = xz + xy\frac{\partial z}{\partial y}$$

And solving for $\frac{\partial z}{\partial y}$ yields: (Mendelson, 1999).

CONCLUDING REMARKS

The aim of this study was to outline certain aspects of partial derivatives as it relates to the surfaces - curves, helixes, etc on the renewable energy and its sustainability. The study highlighted that partial derivatives can be applied on the surfaces of revolution, tangents, plane surfaces and curves which have wider applications on geophysical phenomena. Hence, for renewable energy and substance to work in area of mathematics (Pure and Applied), partial derivatives of functions must be given priority if the entire science is to be enhanced experimentally through manpower training and adequate workshops organized. Partial derivatives is highly essential in the area of mathematics - analysis, algebra, trigonometry and topology, and should be given due attention.

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