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Proportion Defective Chart to monitor ARSAN Sachet Water Production Process

***Nse Okon Bassey**
Moffat Victoria Ekpedeme
James Innocent I.
Department of Statistics, Akwa Ibom State Polytechnic, Ikot Osurua
**E-mail: nse.bassey@akwaibompoly.edu.ng*

ABSTRACT

This study applies the proportion defective control chart (p-chart) to monitor the fraction of non-conforming or defective bags of sachet water produced by ARSAN Water Company in Ikot Ekpene. The aim is to inform the ARSAN water company on how to detect defective bags in the production process. The variation in the production was examined for seventy-six (76) days within the peak period of sales (from February to April, 2024). A sample of 100 bags of sachet water was taken each day for the 76 days, and the number of leaked (defective) bags of water was ascertained in each sample from the production department. The p-chart, Upper Control Limit (UCL), and the Lower Control Limit (LCL) for the proportion of defective bags in the 76 day sample of 100 in each case were constructed. The sample proportion defective remained within the upper and the lower control limits of the p-chart, except for the two sample proportion defectives submitted for days 32 and 76, which were out of control. This indicates that the production process is somewhat out of control. However, on average, there were about 5.7% defective bags of sachet water every day, indicating a high proportion of defectives. The ARSAN Water Company's process with the defective bags is not acceptable. Hence, the production unit should employ other quality inspection and assurance procedures to minimize the number of defective bags in the production process.

Keywords: *Proportion defective, p-chart, upper control limit, lower control limit, production process control*

INTRODUCTION

In an organization, product quality management ensures that customers obtain products that are defect-free. Each finished product has one or more quality characteristics that could be inspected and analyzed during the production process. ARSAN water is not left out in this aspect. Safe and readily available water is for public health, whether it is used for drinking, domestic, food production, or recreation. Water is an essential human nutrient. It is used directly as drinking water or indirectly as a constituent of food. The need to define the quality of water has developed with the increasing demand for water, which is suitable for specific uses, and the creation of the desired quality (Deborah, 1999). No one will purchase water of low quality; hence, it is obvious for ARSAN to manufacture water that is of high quality.

ARSAN Water Company is a major producer of clean and safe drinking water. Hence, the company's output goes through a quality control process to ensure sales. Each finished item has one or more quality characteristics that must be inspected. The ratio of the number of defective items in the population is called the **proportion defective**. The produce is considered **defective** if at least one of the characteristics does not meet the requirements. This could be caused by the operators, supervisors, packers, checkers, or inspectors. Quality must be monitored during the production process (Brahams, 2017). **Quality control** is defined as the process of evaluating goods for quality after they have been produced and sorted into appropriate and unacceptable categories (Cupta & Hira, 2014). Variations are unavoidable in any process, since no process can be run in perfect condition for a period (Dunchan, 1986). The control charts aid in the management and recognition of certain variables and defectives (Montgomery, 2000).

Hence, this study applies the proportion defective control chart (p-chart) to monitor the fraction of non-conforming or defective bags of sachet water produced by ARSAN Water Company in Ikot Ekpene, with the aim of informing ARSAN Water Company on how to detect the defective bags in the production process.

2.0 METHOD

The study adopted a descriptive research design and focused on the quality control process of the ARSAN Water Company, located along Aba Road in Ikot Ekpene. The secondary data were extracted from the production record of ARSAN Water Company. The production process of ARSAN was observed from February to April 2024. Daily production, along with the number of defects, was recorded. There were a few quality-related problems, such as material, production, colour, sealing, machines, and human, among others. The study considered machine and material defects. Finished bags of water that do not have leakages were considered non-defective. Records of production for 76 days were obtained with a sample of 100 bags each day. The collected data were tabulated and analysed using Ms-Excel. The proportion defective (p-chart) is used to assess if the production process is under control.

2.1 Proportion Defective Chart (P-Chart)

P-charts are used to measure the proportion of items in a sample that are defective.

P-charts are suitable when both the number of defectives measured and the size of the total sample are discrete.

2.2 **Assumption**

(i) ARSAN water manufacturing process is in good working order.

(ii) Let π be the assumption that a certain unit will not conform to specification. The fraction (proportion) is expressed as a decimal or as a percentage.

(iii) Successive bags of water produced are independent.

(iv) Each bag of water produced follows a Bernoulli distribution with parameter "π".

Derivation of Parameters:

Let n be a random sample of selected bags of sachet water produced and M be the number of bags that are defective. Then M follows a binomial distribution with parameters "n and π" such that P (M=y) = $\binom{n}{y}\pi^{y}\,(1-\pi)^{n-y}$; y=o, 1,…, n) where n is the combination of n items taken y at a time; the mean of the variable M is $n\pi$ and the variance is $n\pi q$ or $n\pi(1-\pi)$.

The sample proportion defective is the ratio of the number of defective items in the sample, "m" to the sample size "n" and is given by $\bar{\pi}_{\iota} = \frac{M}{n}$.

The mean of this estimator is $\mu=\bar{\pi}$ and the variance is $\sigma_{\pi}^{2} = \bar{\pi}\ (1-\bar{\pi})/n$.

If the true proportion conforming π is known, then the control limits of the P – chart is given by

$$UCL = \bar{\pi} + 3\sqrt{\frac{\bar{\pi}(1-\bar{\pi})}{n}}$$

$$CL = \bar{\pi}$$

$$LCL = Max\ [0, \bar{\pi} - 3\sqrt{\frac{\bar{\pi}(1-\bar{\pi})}{n}}$$

If the proportion π is unknown it can be estimated from the available data. The data is obtained by selecting "m" preliminary samples each of size n.

If there are M_i defectives in sample "I" the proportion defective in simple "i" is $\hat{\pi}_i = \frac{M_i}{n}$, where i = 1, 2, …, m and the average of these individuals sample fractions is

$$\bar{\pi} = \frac{\sum_{i=1}^{m} M_i}{mn} = \frac{\sum_{i=1}^{n} \hat{\pi}_i}{m}$$

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RESULTS

The CL for P-chart were computed for 76 samples with a sample size of 100 each. The estimated average number of defective bags of sachet water was calculated as

$$\hat{\mu}_{\pi} = \frac{\sum_{i=1}^{m} \widehat{M}_i}{m} = \frac{430}{76} = 5.66 \text{ with a standard deviation 2.63.}$$

The average proportion defective was calculated as π.

$$\bar{\pi} = \frac{\sum_{i=1}^{m} M_i}{mn} = \frac{430}{76 \, x \, 100} = \frac{430}{7600} = 0.057$$

$$\text{Or } \pi = \frac{\sum_{i=1}^{m} \pi_i}{m} \quad \frac{34}{76} = 0.057$$

The control limits are calculated as follows:

$$\text{UCL} = \bar{\pi} + 3\sqrt{\frac{\bar{\pi}(1-\bar{\pi})}{n}} = 0.057 + 3\left(\sqrt{\frac{o.057\ (1-0.057)}{100}}\right) = 0.126$$

$$\text{LCL} = \bar{\pi} - 3\sqrt{\frac{\bar{\pi}(1-\bar{\pi})}{n}} = \sqrt{\frac{0.057(1-0.057)}{100}} = -0.013$$

$$= \mathbf{0}$$

Table 1: Calculation of proportion defectives and control limits

S/N	SAMPLE	No. of defective	proportion of defective (p)	Average proportion of defective	UCL	LCL
1	100	10	0.1	0.057	0.126	0.000
2	100	5	0.05	0.057	0.126	0.000
3	100	3	0.03	0.057	0.126	0.000
4	100	7	0.07	0.057	0.126	0.000
5	100	5	0.05	0.057	0.126	0.000
6	100	4	0.04	0.057	0.126	0.000
7	100	4	0.04	0.057	0.126	0.000
8	100	7	0.07	0.057	0.126	0.000
9	100	6	0.06	0.057	0.126	0.000
10	100	6	0.06	0.057	0.126	0.000
11	100	8	0.08	0.057	0.126	0.000
12	100	9	0.09	0.057	0.126	0.000

13	100	4	0.04	0.057	0.126	0.000
14	100	5	0.05	0.057	0.126	0.000
15	100	7	0.07	0.057	0.126	0.000
16	100	3	0.03	0.057	0.126	0.000
17	100	7	0.07	0.057	0.126	0.000
18	100	6	0.06	0.057	0.126	0.000
19	100	10	0.1	0.057	0.126	0.000
20	100	7	0.07	0.057	0.126	0.000
21	100	6	0.06	0.057	0.126	0.000
22	100	3	0.03	0.057	0.126	0.000
23	100	2	0.02	0.057	0.126	0.000
24	100	2	0.02	0.057	0.126	0.000
25	100	5	0.05	0.057	0.126	0.000
26	100	7	0.07	0.057	0.126	0.000
27	100	4	0.04	0.057	0.126	0.000
28	100	8	0.08	0.057	0.126	0.000
29	100	6	0.06	0.057	0.126	0.000
30	100	10	0.1	0.057	0.126	0.000
31	100	11	0.11	0.057	0.126	0.000
32	100	3	0.03	0.057	0.126	0.000
33	100	8	0.08	0.057	0.126	0.000
34	100	2	0.02	0.057	0.126	0.000
35	100	1	0.01	0.057	0.126	0.000
36	100	8	0.08	0.057	0.126	0.000
37	100	3	0.03	0.057	0.126	0.000
38	100	9	0.09	0.057	0.126	0.000
39	100	8	0.08	0.057	0.126	0.000
40	100	4	0.04	0.057	0.126	0.000
41	100	5	0.05	0.057	0.126	0.000
42	100	2	0.02	0.057	0.126	0.000
43	100	4	0.04	0.057	0.126	0.000
44	100	3	0.03	0.057	0.126	0.000
45	100	8	0.08	0.057	0.126	0.000
46	100	5	0.05	0.057	0.126	0.000
47	100	2	0.02	0.057	0.126	0.000
48	100	8	0.08	0.057	0.126	0.000
49	100	8	0.08	0.057	0.126	0.000
50	100	7	0.07	0.057	0.126	0.000
51	100	9	0.09	0.057	0.126	0.000
52	100	3	0.03	0.057	0.126	0.000
53	100	1	0.01	0.057	0.126	0.000
54	100	8	0.08	0.057	0.126	0.000
55	100	5	0.05	0.057	0.126	0.000
56	100	6	0.06	0.057	0.126	0.000
57	100	5	0.05	0.057	0.126	0.000
58	100	11	0.11	0.057	0.126	0.000
59	100	10	0.1	0.057	0.126	0.000
60	100	3	0.03	0.057	0.126	0.000
61	100	4	0.04	0.057	0.126	0.000

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62	100	7	0.07	0.057	0.126	0.000
63	100	2	0.02	0.057	0.126	0.000
64	100	9	0.09	0.057	0.126	0.000
65	100	6	0.06	0.057	0.126	0.000
66	100	4	0.04	0.057	0.126	0.000
67	100	7	0.07	0.057	0.126	0.000
68	100	11	0.11	0.057	0.126	0.000
69	100	3	0.03	0.057	0.126	0.000
70	100	2	0.02	0.057	0.126	0.000
71	100	4	0.04	0.057	0.126	0.000
72	100	7	0.07	0.057	0.126	0.000
73	100	5	0.05	0.057	0.126	0.000
74	100	3	0.03	0.057	0.126	0.000
75	100	3	0.03	0.057	0.126	0.000
76	100	7	0.07	0.057	0.126	0.000
Total		430	4.3			

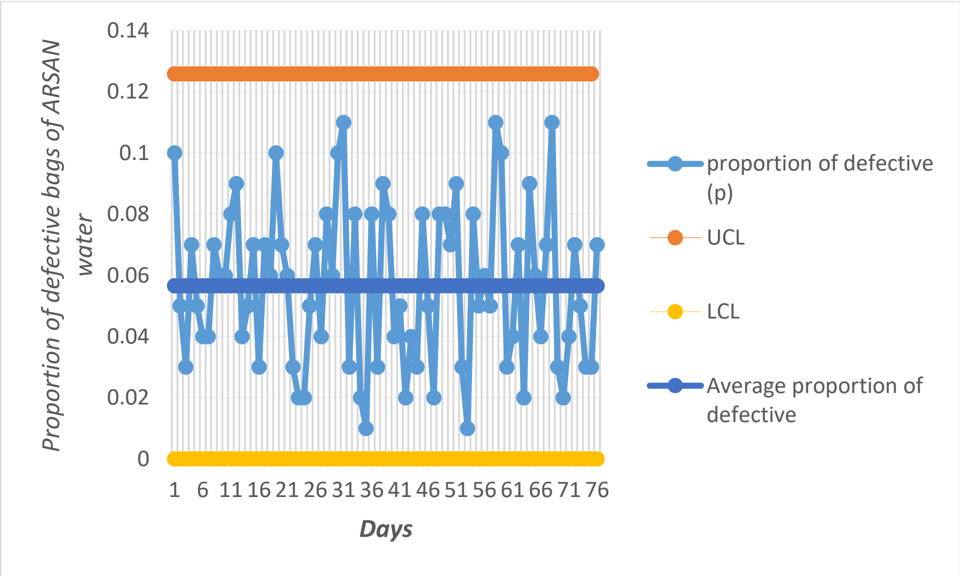

Figure 1: P-Chart

The P-chart plots the proportion of defectives against sample number. The centre line of the graph is given by which the defect rate from all samples is contained. Control lines for samples are plotted along the upper control limit and the lower control limit. The variation in the production of sachet water was examined from day 1 to 76. The P-chart reveals that the production process was out of statistical control. This means that production was inconsistent and unpredictable. On average, each day, there was about 5.7% defective bags. Similarly, on a given day, exactly 2 sample proportion defectives lie outside the UCL, revealing that the production process is out of statistical control. Also, 5.7% defective bags of sachet water are not acceptable in the production process. Therefore, the production unit should employ other quality inspection tools and methods to minimize the number of defectives during the production process.

CONCLUSION

The sachet water was the finished product of ARSAN Water Company, considered for the study. The number of bags produced was examined, and the fraction of water conforming was observed. The proportion control chart was used to inspect the defectives. It was found that the proportion of defectives is out of control limits; hence, the production process of ARSAN Water Company is out of statistical control. Nonetheless, the production department should also aim at enhancing productivity in order to ensure consistency in the quality of ARSAN water production. Furthermore, appropriate records of production should be kept to ease data collection.

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