GRAVITY GRADIENT SATELLITE STABILISATION WITH ACTIVE DAMPER USING OPTIMAL CONTROL

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ABSTRACT

The major challenge of passive attitude control for small satellite are the lack of accuracy and the inability to effectively damp oscillations associated with attitude reorientation due to presence of external disturbances on the satellite. To reduce these undesired oscillations, passive dampers may be used, but they take a long time to achieve required results. A study of an active damper that applies a magnetic torque was presented as an alternative control strategy of small satellites to produce the needed moments to counteract the external disturbances. Linear Quadratic Gaussian (LQG) controller was designed for the attitude control of small satellite, incorporating active magnetic damping. Results showed that both attitude reorientation and active damping were achieved in less than 30s.

***Keywords:** Linear Quadratic Gaussian, Small Satellite, Satellite Attitude Control, Dampers, Band-Limited Noise,*

INTRODUCTION

Satellites rapidly grew increasingly large and enormously expensive after the first launch in 1957 which limited access to only a relatively few nations. However, the development in micro-electronics stimulated by consumer market rather than military and possibility of constellations of small satellites to provide a range of real-time global services have brought challenges for affordable small satellites. Nevertheless, this comes at the cost of less powerful sensors and actuators, as well as reduced computational power, due to size and weight limitations (Sidi, 1997). Among other sub-systems, the attitude determination and control system was affected by this trade-off, leading to more challenging attitude control and determination problems as explained by Robert, Pedro T. and Pedro L. (2000), who found in low earth orbit about 1,200km above the surface of the earth. In order to achieve continuous satellite access, a larger network of satellites is required with regular connection handover between them, termed formation flight (Timothy and Charles, 1986). It is therefore vital for them to overcome any disturbances that can disrupt their mission hence, the challenge for attitude control. The classification of satellites according to mass as referenced by Gottfried (2004) is shown on Table 1.

Table 1: *Classification of Satellites According to Mass*

Group Name	Mass	
Large Satellite	>1000kg	
Medium Satellite	500-1000kg	
Mini Satellite	100-500kg	Small Satellites
Micro Satellite	10-100kg	
Nano Satellite	1-10kg	
Pico Satellite	0.1-1kg	
Femto Satellite	<100g	

The rest of this paper is arranged as follows: Section 2 gives the Satellite Attitude Control, Section 3 presents the Satellite Attitude System Model, and Section 4 presents the Results and Discussion of Results while Section 5 concludes the paper.

SATELLITE ATTITUDE CONTROL

The control of satellite attitude since the launching of the first satellite in late 1950's has been one of the several most studied space research areas (Ouhocine, Filipski, Mohd, and Ajir, 2004). The control of satellite in orbit could be single- and dual-spin stabilisation, gravity gradient stabilisation, thrusters, magnetic control device and wheels, constituting either passive or active control system (Sidi, 1997) as shown in Figure 1.

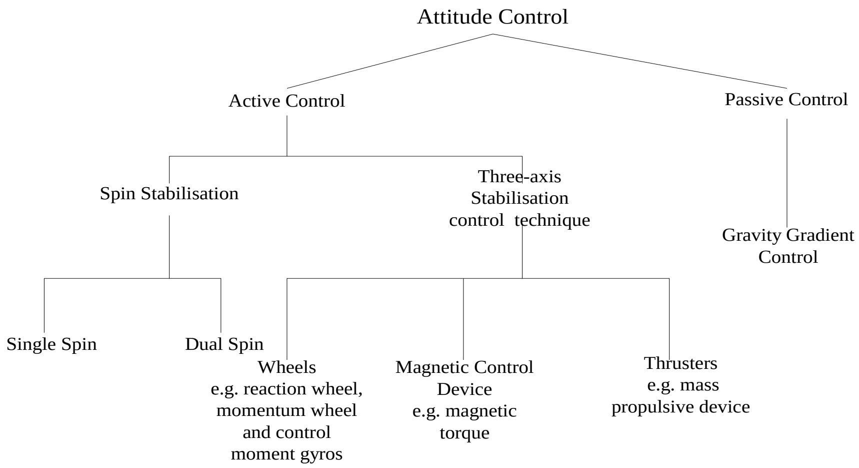

Figure 1: *Classification of Satellite Attitude Control*

The passive attitude control method uses the natural physical properties of the satellite and its environment to control the satellite attitude. It is attractive for the small satellite because the hardware required is less complicated and relatively inexpensive with lower accuracies than those that are possible with active attitude control, which uses sophisticated and more expensive control instrumentation.

Gravity Gradient Stabilisation: Gravity gradient stabilisation is a passive attitude control method that relies on satellite mass distribution for attitude control (Ouhocine, Filipski, Mohd and Ajir, 2004; Gilberto, Luiz and Adrielle, 2009). It uses the change in gravity with altitude to create a torque when the principal axes are not aligned with the orbital reference frame. Long booms are usually extended to create the torque. Due to the asymmetric nature of the satellite, the satellite being subjected to the earth gravitational field will experience a torque tending to align its axis of least inertia with the field direction (Kristin, 2001). However, the relative values of moment of inertia of the satellite about the centre of mass, the orbital rate and the presence of external disturbance on the satellite cause the satellite to oscillate (Sidi, 1997). The gravity gradient provides the restoring or stabilising torque but does not damp the oscillation.

Dampers: Dampers are devices used to control oscillation. A common and cheap method used to reduce the undesired oscillations in the gravity gradient stabilisation of a satellite is by the use of passive dampers (Fleeter and Warner, 1989), even though the time to appreciably decrease the oscillatory motion might be very long. For this reason active dampers, like the magnetic torqrods have been used in the control system of satellite. These interact with the earth's magnetic field to produce the needed moments to counteract external disturbances to the satellite. Ouhocine, Filipski, Mohd and Ajir (2004) compared passive and active dampers for a gravity-gradient stabilised small satellite attitude control methods. They designed a Proportional-Derivative (PD) control algorithm used to damp the satellite oscillations around its equilibrium position. This work use optimal control technique to design a Linear Quadratic Gaussian (LQG) controller for a gravity gradient stabilised satellite.

Optimal Controller: In optimal control, one attempts to find a controller that provides the best possible performance with respect to some given measure of performance (Roland, 2001). That is, a controller that uses the least amount of control-signal energy to drive the output to zero within a short possible time (Kristin, 2001). Gilberto, Luiz and Adrielle (2009) used LQG technique to design a feedback control law with a Pulse-Width Pulse-Frequency (PWPF) modulator to reject external disturbances affecting a Brazilian satellite and regulate the performance of a satellite attitude control using reaction thrusters. Also Maria, Benedito, Simplício and Silva (2004) analysed the responses of a conventional PID and an optimal LQG controllers designed for an electromagnetic dynamometer. The settling time for the LQG was shorter than that of the PID. Sethi and Song (2005) designed a full state feedback LQR and an observer to implement an active controller for vibration suppression in a model frame structures.

The simulation results confirmed a convergence of the plant output in a few tenths of a second. Leila, Habibnejad and Amin (2008) developed an optimal controller for a two link-robotic manipulator systems using an LQG by Kalman that proved effective for a state space dynamics of the systems. In the above literatures, synthesis of control laws by either LQR or LQG were characterised by simplicity on implementation with effective performances.

Linear Quadratic Gaussian (LQG): The LQG control is a modern state space technique for designing optimal dynamic regulators. It enables a trade off in performance and control effort, and takes into account process and measurement noise. It consists of an optimal state-feedback gain and a state estimator gain. The first design step is to seek a state-feedback law that minimizes the cost function of regulation performance, which is measured by a quadratic performance criterion with user-specified weighting matrices, *Q* and *R* or design parameters (Leila, Habibnejad and Amin, 2008; Gilberto, Luiz and Adrielle, 2009). The second step is to design the state estimator which can be done using the Kalman estimator or by standard pole assignment techniques. The standard pole placement techniques according to the work of Sethi and Song (2005) is utilised in this work to design the estimator.

The dynamics of the estimator due to the presence of process and the measurement noise in the system are given by

$$\dot{\hat{x}} = A\hat{x} + Bu + L\left(y - \hat{y}\right)$$
$$\hat{y} = C\hat{x} \quad \text{...1}$$

where $\hat{x}$ is the estimate of x, $\hat{y}$ is the estimate of y and L is the estimator gain that determine the convergence of $\hat{x} \to x$.

The state estimation error and its derivative is given as

$$\varepsilon = x - \hat{x}$$
$$\dot{\varepsilon} = \dot{x} - \dot{\hat{x}} \quad \text{...2}$$

The estimator gain can be obtained using the conventional Ackermann formula by substitution.

$$\dot{x} - \dot{\hat{x}} = A\left(x - \hat{x}\right) - L\left(Cx - C\hat{x}\right) \quad \text{...3}$$

Hence,

$$\dot{\varepsilon} = \left(A - LC\right)\varepsilon \quad \text{...4}$$

If the observer is in a closed-loop system, the optimal controller will therefore be modified as

$$u = -K\hat{x} \quad \text{...5}$$

Now, ε converges to zero as long as $A - LC$ is asymptotically stable. It turns out that even when A is unstable, proper selection of L will make $A - LC$ asymptotically stable.

The closed-loop system can be written as

$$\dot{x} = Ax - BK\hat{x} = (A - BK)x + BK(x - \hat{x})$$
$$\dot{\varepsilon} = (A - LC)\varepsilon \qquad ...6$$

This is written in matrix form as

$$\begin{bmatrix} \dot{x} \\ \dot{\varepsilon} \end{bmatrix} = \begin{bmatrix} (A - BK) & BK \\ 0 & (A - LC) \end{bmatrix} \begin{bmatrix} x \\ \varepsilon \end{bmatrix} \qquad ...7$$

The schematic diagram of the close-loop control system is shown in Figure 2.

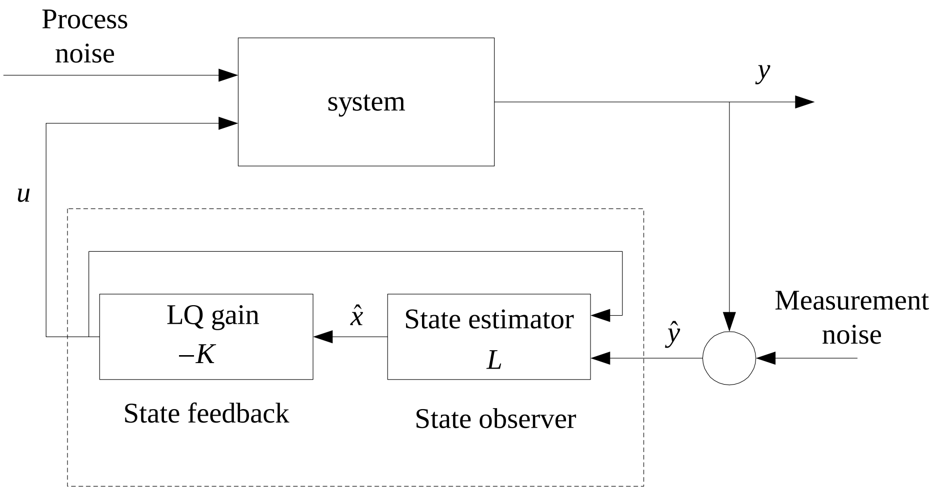

Figure 2: *The schematic diagram of the closed-loop control system showing the LQG controller*

The eigenvalues of the above matrix are the eigenvalues of $(A - LC)$ and the eigenvalues of $(A - BK)$. Through the separation principle, the optimal gain matrix K and the observer gain matrix L can be selected independently.

Therefore, to effectively suppress the associated vibration with the gravity gradient stabilised small satellite it is necessary to choose the weighting matrices Q and R appropriately as they represent the design parameters for the controllers.

SATELLITE ATTITUDE SYSTEM MODEL

The attitude model of the satellite includes both the kinematics and the dynamics of the satellite. The kinematics defines entirely the change in the orientation of the satellite irrespective of the forces acting on the satellite, while the dynamics defines the time dependent parameters as a function of the external forces acting on the satellite.

The attitude dynamics equation can be analysed using the operator (Sidi, 1997) in equation 8:

$$\dot{A}\big|_i = \dot{A}\big|_b + \omega \times A \qquad \text{...8}$$

This states that the rate of change of a vector A as observed in a fixed reference frame (inertial frame) equals the rate of change of the vector as observed in a rotating coordinate system (body frame) with angular velocity ω, plus the vector product $\omega \times A$. The rotational equations for a rigid body are derived with the rotational equivalent of:

$$\dot{\bar{h}} = \bar{g} \qquad \text{...9}$$

where $\bar{h}$ is the angular momentum about the mass centre, and $\bar{g}$ is the torque (gravity gradient and magnetic - we considered only gravity-gradient torque and magnetic torque in this work for satellite attitude control). This relationship can be represented in matrix form using equation (8) as:

$$\dot{\bar{h}} + \omega_{bi}^{b} \times \bar{h} = \bar{g} \qquad \text{...10}$$

The linearised satellite attitude equation for a three-axis stability according to Kristin (2001) and Wisniewski (1996), for the satellite to always point to the earth with its nadir vector, given in state space form is:

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} = A \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} + \frac{-I_m^{-1}}{\|B_m\|} B_m \times B_m \times \tilde{M} \qquad \text{...11}$$

B_m is the geomagnetic field in the satellite body frame and $\|B_m\|$ is its norm. I_m is the moment of inertia of the satellite and $\tilde{M}$ *is the control torque*, $(\tilde{M} = u)$
where

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -4\omega_c^2 r_1 & 0 & 0 & 0 & 0 & \omega_c - \omega_c r_1 \\ 0 & 3\omega_c^2 r_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_c^2 r_3 & -\omega_c - \omega_c r_3 & 0 & 0 \end{bmatrix} \qquad \text{...12}$$

$$r_1 = \frac{I_{yy} - I_{zz}}{I_{xx}}, \quad r_2 = \frac{I_{zz} - I_{xx}}{I_{yy}}, \quad r_3 = \frac{I_{xx} - I_{yy}}{I_{zz}} \qquad ...13$$

I_{xx}, I_{yy} *and* I_{zz}, are the moments of inertia about the axes of the body frame and ω_c is the orbital angular velocity.
The input control matrix is:

$$B = I_m^{-1} * G \qquad ...14$$

where

$$G = \begin{bmatrix} \left(B_2^2 + B_3^2\right)/B^2 & -B_1B_2/B^2 & -B_1B_3/B^2 \\ -B_1B_2/B^2 & \left(B_1^2 + B_3^2\right)/B^2 & -B_2B_3/B^2 \\ -B_1B_3/B^2 & -B_2B_3/B^2 & \left(B_1^2 + B_2^2\right)/B^2 \end{bmatrix} \qquad ...15$$

B_1, B_2 and B_3 are components of the earth magnetic field in the satellite body frame. The off diagonal terms of G have an average value of zero while the diagonal terms, defined as g_x, g_y *and* g_z respectively have average values that are a function of orbit inclination. The dependence according to Barry (2003) is shown on Table 2.

Table 2: *Average components of Earth Geomagnetic Field against Inclination at 560km: IGRF 2000 (Ref.:* [1]*).*

Inc. (degree)	g_x	g_y	g_z	*Inc. (degree)*	g_x	g_y	g_z
0	0.967	0	0.804	*60*	0.739	0.857	0.39
10	0.995	0.068	0.781	*70*	0.709	0.923	0.353
20	0.922	0.256	0.711	*80*	0.691	0.965	0.335
30	0.876	0.46	0.614	*90*	0.686	0.981	0.333
40	0.826	0.632	0.522	*100*	0.691	0.965	0.335
50	0.78	0.762	0.446	*110*	0.709	0.923	0.353

The control law was tested by performing simulations with satellite configurations and initial conditions obtained from Ouhocine, Filipski, Mohd and Ajir (2004). The control tuning matrices *R* and *Q* were obtained through iterative process following expectable requirements such that the system damps to the desired equilibrium within limited time with allowable control effort. Also the measurement noise was modelled as band-limited white noise for the simulation. Figure 3 and table 3 show the response of the measurement noise and the noise statistics used in the simulation respectively, while table 4 shows the satellite data obtained from Ouhocine, Filipski, Mohd and Ajir (2004) for the simulation.

Table 3: Noise Statistics

Parameter	Measurement Noise
Noise power	0.00001 rad
Covariance	0.0002 $(rad)^2$
Sample time	0.05 s
Bandwidth	1.26 rad/s
Mean	0 rad

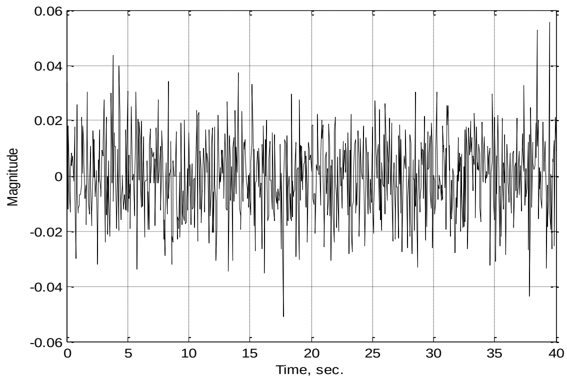

Figure 3: Measurement noise response

Table 4: Satellite Characteristics (From: [8]).

Moment of inertia I_x	100kg/m^2
Moment of inertia I_y	100kg/m^2
Moment of inertia I_z	2.5kg/m^2
Altitude h	560km
Inclination β	60 degree
Orbital rate ω_c	0.0010764 rad/sec
Desired Euler angles $[\phi, \theta, \varphi]$	$[0, 0, 0]$ rad
Simulation Time	40 seconds

Source: Ouhocine, Filipski, Mohd and Ajir (2004)

The initial conditions for roll pitch and yaw, and their rates as well as the weight matrices used in the simulations are shown in SET 1 and SET 2 with their respective results.

***SET 1*:**

$$[\phi \;\; \theta \;\; \varphi] = [1.3963 \quad 1.0472 \quad \text{-}1.3963\ ]\,rad. \;\; \equiv \;\; (80,\ 60,\ \text{-}80)\deg.$$

$$[\dot{\phi} \;\; \dot{\theta} \;\; \dot{\varphi}] = [0.0005 \;\; 0.0003 \;\; \text{-}0.003]\,rad\,/\,\text{sec}.$$

$$Q = diag([100\ 100\ 100\ 0.01\ 0.01\ 0.01])$$

$$\text{R} = \text{I}_{3\times3}$$

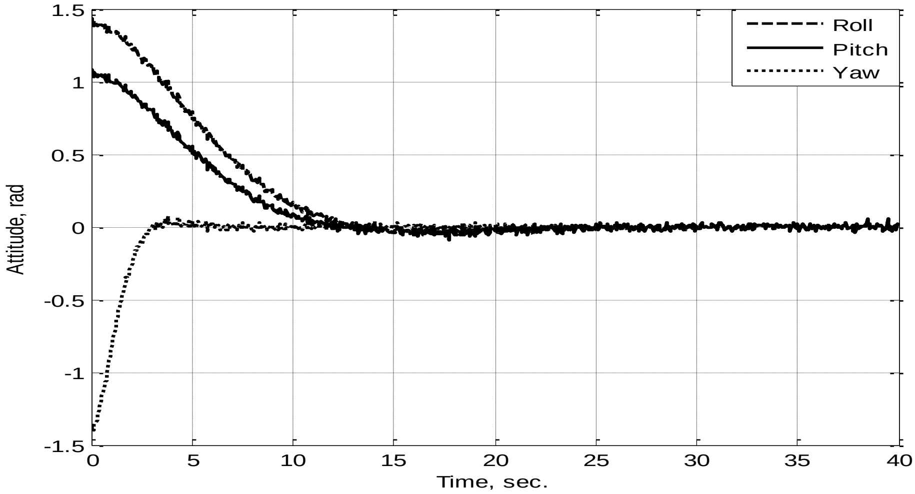

Figure 4(a): Simulation responses for Estimated output

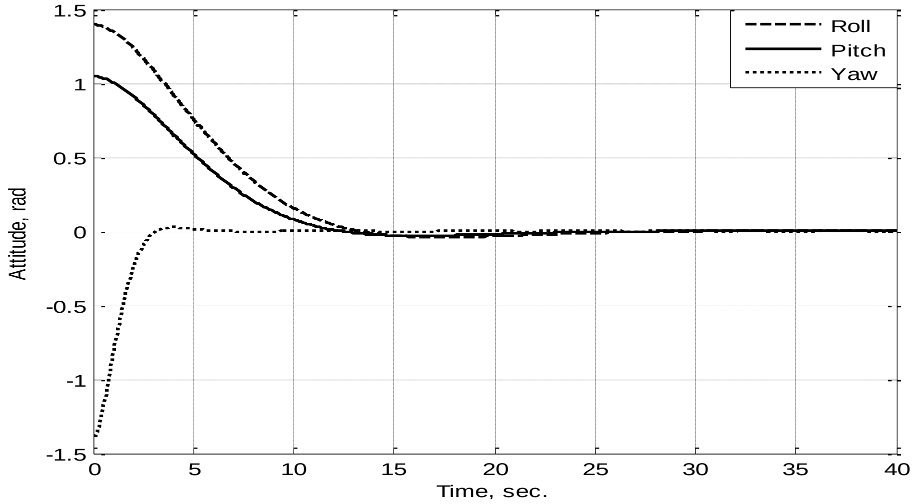

Figure 4(b): Simulation responses for LQG controller

***SET 2*:**

$$\left[\phi \;\; \theta \;\; \varphi\right] = \left[0.0524 \quad 0.0175 \;\; \text{-}0.0524\right] rad. \;\; \equiv \;\; \left(3, 1, \text{-}3\right) \text{deg}.$$

$$\left[\dot{\phi} \;\; \dot{\theta} \;\; \dot{\varphi}\right] = \left[0.0005 \;\; 0.0003 \;\; \text{-}0.003\right] rad/\text{sec}.$$

$$Q = diag\left(\left[100\ 100\ 100\ 0.01\ 0.01\ 0.01\right]\right)$$

$$\text{R} = \text{I}_{3\times3}$$

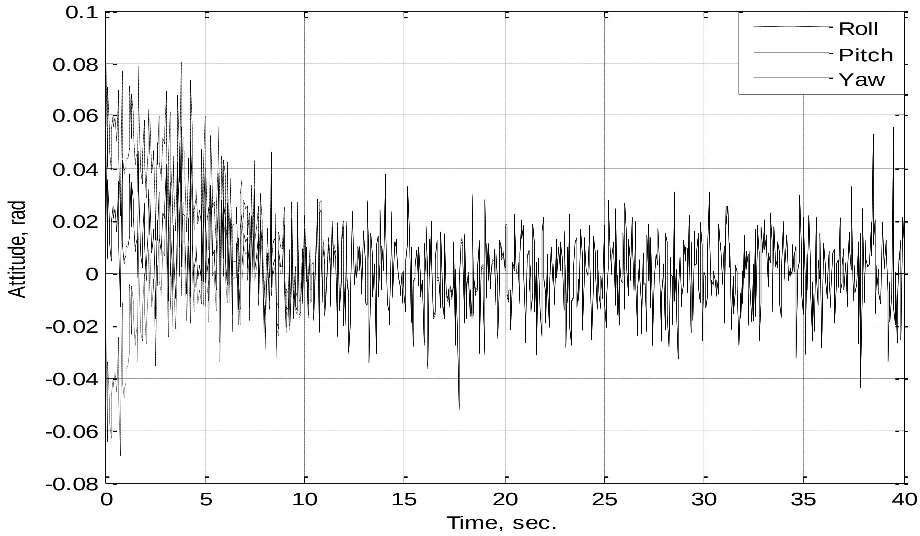

Figure 5(a)*: Simulation responses for Estimated output,*

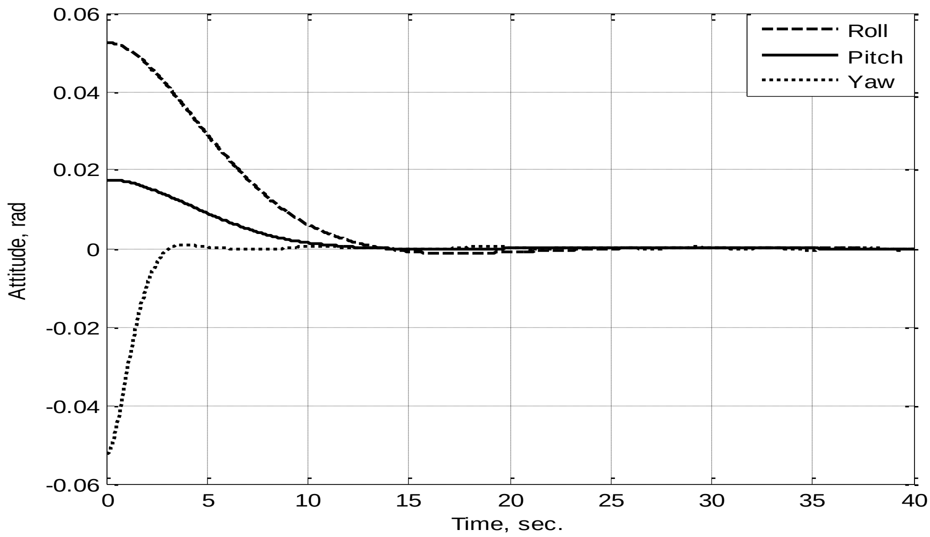

Figure 5(b): Simulation responses for LQG controller

It is observed that the LQG controller was able to damp the oscillations in the system even for large initial attitude displacements within a short period of time as possible. Figures 4(b) and 5(b) are the estimated outputs due to introduction of the measurement noise (Figure 3) to the system. The noise was filtered out by the state estimator of the controller as shown in Figures 4(a) and 5(a) with settling time of 25s, 20s and 7s in the roll axis, pitch axis and yaw axis respectively with a maximum control torque of 1.55E-6Nm. Hence, the higher the angular displacement in the satellite attitude the higher the magnetic control torque needed to damp the oscillations. This also shows the robustness of the controllers to attitude parameter changes and also to quickly restore the orientation of the satellite for efficient communication with the base station. The responses of Figures 4-5 show better performances in the use of optimal controllers when compared to the solution of Ouhocine, Filipski, Mohd and Ajir (2004) where they used PD as an active method for controlling the attitude of the small satellite.

CONCLUDING REMARK

The use of active damper to control a gravity gradient stabilised small satellite is presented in this work. A linearised satellite attitude dynamics in state-space form was used for the application of the designed optimal controller- Linear Quadratic Gaussian (LQG). After many simulations with different initial conditions of angular displacement, weighting matrices and observer gain matrix, the controller was able to damp the gravity gradient oscillation within a short period of time.

REFERENCES

Barry, S. L. (2003). NPSAT1 Magnetic Attitude Control System. 16th Annual AIAA/USU Conference on Small Satellites. Washington, DC. pp.1-7.

Fleeter, R. and **Warner, R.** (1989), *Guidance and Control of Miniature Satellites, Automatic Control in Aerospace, IFAC* (Tsukuba, Japan), Oxford, UK: Pergamon. pp. 243-248.

Gilberto, A., Jr., Luiz, S. M. and **Adrielle, C. S.** (2009). Optimal On-Off Attitude Control for the Brazilian Multimission Platform Satellite. Hindawi Publishing Corporation Mathematical Problems in Engineering Volume 2009, Article ID 750945. 17 pages.

Gottfried, K. (2004). Small Satellites - A Tool for Earth Observation. Invited Paper: Commission IV, Institute of Photogrammetry and GeoInformation, University of Hannover, Hannover.

Kristin, L. M. (2001). A Nonlinear Magnetic Controller for Three-Axis Stability of Nanosatellites. Unpublished Master Thesis, Department of Aerospace Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia.

Leila, F. A., Habibnejad, M. K. and **Amin, N.** (2008). Linear-Quadratic-Gaussian Controller for Two link-robotic Manipulator Control. In: Proceedings of the World Congress on Engineering and Computer Science, WCECS 2008, San Francisco, USA.

Maria, L. G., Benedito, S. O., Simplício, A. S. and **Silva, A.** (2004). Conventional and LQG Controllers applied to an Electromagnetic Dynamometer. ABCM Symposium Series in Mechatronics. Department of Mechanical Technology - CT - UFPB - Campus I - CEP:58059-900 - João Pessoa - PB, Brazil Vol. 1. PP 206-212.

Ouhocine, O., Filipski, M. N., Mohd, N. S. B. and **Ajir, M. R.** (2004). Small Satellite Attitude Control and Simulation. Jurnal Mekanikal, Bil. 17: 36-47.

Robert, C., Pedro, T. and **Pedro, L.** (2000). Small Satellite Attitude Control Based on a Kalman Filter. In: Proceeding of IEEE on intelligent control. Patras, Greece, pp.7-14.

Roland, S. B. (2001). Advanced Control Engineering, Butterworth. Heinemann Publishers. pp.232-260 and pp.272-317.

Sethi, V. and **Song, G.** (2005). Optimal Vibration Control of a Model Frame Structure Using Piezoceramic Sensors and Actuators. Journal of Vibration and Control. pp.671-684, 2005.

Sidi, M. J. (1997), Spacecraft Dynamics and Control: A Practical Engineering Approach. Cambridge University Press, pp.89-97and pp.112-130.

Timothy, P. and **Charles, W. B.** (1986), Satellite communications. John and Wiley and Sons, New York. pp.11-33 and pp.55-66.

Wisniewski, R. (1996), Satellite Attitude Control Using Only Electromagnetic Actuation. Unpublished PhD Thesis, Department of Control Engineering Aalborg University.