# Effects of Radiation and Radial Magnetic Field on Steady Free Convection Flow in a Vertical Porous Concentric-Annular due to Convective Surface Boundary Condition

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## ABSTRACT

*The problem concerning the effects of radiation and radial magnetic field on steady free convection flow in a vertical porous concentric-annular due to convective surface boundary condition has been studied. The models governing equations are solved by using perturbation method. The results show that the maximum flow velocity and temperature are recorded at the lower plate by increasing the symmetric wall temperature while opposite phenomenon is observed at the upper plate. The parameters such as ambient temperature parameter, magnetic parameter, Biot number as well as convective heat transfer parameters have effects on temperature and velocity.*

***Keywords**: Heat and mass transfer, mixed convection, differential transform method, exothermic chemical reaction, and vertical channel*

## NOMENCLATURE

| **Description**<br>**Symbol** | |
|---|---|
| Prandtl number | Pr |
| Magnetic parameter | M |
| Radiation parameter | R |
| Suction/injection parameter | $\lambda$ |
| Porous parameter | $K$ |
| Convective heat transfer parameter at y = 0 | $Bi_1$ |
| Convective heat transfer parameter at y = 1 | $Bi_2$ |
| Strength of magnetic field | $B_0$ |
| Ambient temperature parameter | $rt$ |
| Gravitation acceleration | g |
| Grashof number based on temperature | Gr |
| Temperature difference parameter | $C_T$ |

## INTRODUCTION

Radiation effect on natural convection magneto hydrodynamic (MHD) flow in a rotating vertical porous channel partially filled with a porous medium (Chuhan and Rastogi, 2010). Gundagami, Sheri, Paul, and Reddy (2013) report unsteady magneto hydrodynamic free convective flow past a vertical porous plate. Hence, the effect of chemical reaction on an unsteady magneto hydrodynamics free convection flow past a vertical porous plate in the presence of suction or injection (Shivaiah and Rao, 2012). Jha, Aina and Ajiya (2015) reveal that unsteady magneto hydrodynamic (MHD) free convection couette flow between vertical porous plates with thermal radiation. According to Jha and Odengle (2016), magneto hydrodynamic (MHD) natural convection flow in a vertical parallel plate micro channel. Radhakrishnamacharya and Maiti (1977) argue that there is chemical reaction effect on natural convection flow between fixed vertical plates with suction and injection. Sharma, P. R., Kumar and Sharma (2011) report the influence of chemical reaction on unsteady magneto hydrodynamic (MHD) free convective flow and mass transfer through viscous incompressible fluid past a heated vertical plate immersed in porous medium in the presence of heat source.

Also, there is influence of thermal radiation on unsteady free convection magneto hydrodynamic (MHD) flow of Brinkman type fluid in a porous medium with Newtonian heating (Farhad, Ilyas, Sami and Sharidan, 2013). Grosan and Pop (2012) investigation shows fully developed mixed convection in a vertical channel filled by a Nanofluid. Wu-Shung, Wei-Siang, Tzu-En and Chung-Gang (2016) observe a flow downward penetration of vertical parallel plates' natural convection with an asymmetrically heated wall. Wu-Shung, Wei-Hsiang and Shang-Hao (2012) state that there exists a natural convection of three dimensional horizontal plates from a steady to an unsteady situation by a CUDA computation platform. Rao and Babu (2016) studied the finite element analysis of radiation and mass transfer flow past semi-infinite moving vertical plate with viscous dissipation.

Idowu, Abdulwaheed, Oyelami and Dada (2014) investigation also shows that heat and mass transfer of magneto hydrodynamic (MHD) and dissipative fluid flow pass a moving vertical Porous plate with variable suction. A study by Parvin, Nasir, Hossainand and Chamkha (2012) reveal the thermal conductivity variation on natural convection flow of water-alumina nanofluid in an annulus. Nagarajue, Murthy and Sai (2013) expose the steady flow of an electrically conducting incompressible micro-polar fluid in a narrow gap between two concentric rotating vertical cylinders with porous lining on inside of outer annulus under an imposed axial magnetic field.

## Effects of radiation and radial magnetic field on steady free convection flow in a vertical porous concentric-annular due to convective surface boundary condition

### *Mathematical formulation*

Consider nonlinear heat transfer of steady state MHD free-convective, viscous, and incompressible and electrically conducting radiations fluid due to combined suction/injection effects with transverse magnetic field, between two infinite vertical parallel porous walls (Figure 1). The flow is assumed to be laminar and fully developed. That is the axial ( $x'$ -direction) velocity depends only on transverse coordinate $y'$ . Following the Boussiinesq's approximation the required governing equations for the present problem are:

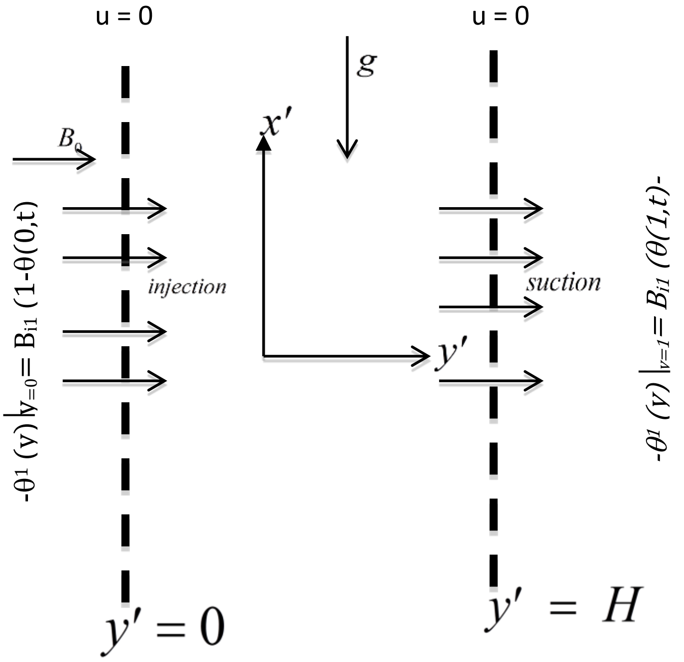

**Figure 1:** Physical configuration of the problem

### Momentum equation

$$\frac{d^2u}{dy^2}-\lambda\frac{du}{dy}-M^2u=-Gr\theta \qquad (1)$$

### Energy equation

$$\left(1+\frac{4R}{3}(C_T+\theta)^3\right)\frac{d^2\theta}{dy^2}+4R(C_T+\theta)^2\left(\frac{d\theta}{dy}\right)^2=-\lambda\,\mathrm{Pr}\frac{d\theta}{dy} \qquad (2)$$

boundary conditions are;

$$\left.\begin{array}{l} u=0,-\dfrac{d\theta}{dy}|_{y=0}=Bi_1(1-\theta(0,t)) \quad at \quad y=0 \\ u=0,-\dfrac{d\theta}{dy}|_{y=1}=Bi_2(\theta(1,t)-rt) \quad at \quad y=1 \end{array}\right\} \qquad (3)$$

### Method of solution

To obtain analytical solution of equation (1) and (2) subject to prescribe boundary condition (3) it is assumed that radiation parameter is small and taking a power series expansion in the radiation parameter (R<<1) apply a regular perturbation method.

Using Perturbation technique we write/Assumed

$$\left.\begin{array}{l} u(y)=\sum\limits_{i=0}^{n}R^iu_i \\ \theta(y)=\sum\limits_{i=0}^{n}R^i\theta_i \end{array}\right\} \qquad (4)$$

$$\frac{d^2\theta_0}{dy^2}-\lambda Pr\frac{d\theta_0}{dy}=0 \qquad (5)$$

$$\frac{d^2\theta_1}{dy^2}-\lambda Pr\frac{d\theta_1}{dy}=-\left(1+\frac{4R}{3}(C_T+\theta_0)^3\right)\frac{d^2\theta_0}{dy^2}-4R(C_T+\theta)^2\left(\frac{d\theta_0}{dy}\right)^2 \qquad (6)$$

and that of momentum equation are:

$$\frac{d^2u_0}{dy^2}-\lambda\frac{du_0}{dy}-M^2u_0=-G_r\theta_0 \qquad (7)$$

$$\frac{d^2u_1}{dy^2}-\lambda\frac{du_1}{dy}-M^2u_1=-G_r\theta_1 \qquad (8)$$

with the following new boundary conditions

$$\left.\begin{aligned}-\frac{d\theta_0}{dy}&=Bi_1(1-\theta_0),-\frac{d\theta_1}{dy}=Bi_1(1-\theta_1),u_0=u_1=0\,at\;y=0\\-\frac{d\theta_1}{dy}&=Bi_2(\theta_0-rt),-\frac{d\theta_1}{dy}=Bi_1(\theta_1-rt),u_0=u_1=0\;at\;y=1\end{aligned}\right\} \qquad (9)$$

The required solution of governing energy and momentum equations (5) to (8) subject to boundary condition (8) is obtained as:

$$\theta(y)=K_2+\frac{K_1}{\lambda\,\mathrm{Pr}}e^{\lambda\mathrm{Pr}\,y}+R(A_3+A_4e^{-\lambda\mathrm{Pr}\,y}+L_1ye^{\lambda\mathrm{Pr}\,y}+L_2e^{2\lambda\mathrm{Pr}\,y}+L_3e^{3\lambda\mathrm{Pr}\,y}+L_4e^{4\lambda\mathrm{Pr}\,y}) \qquad (10)$$

$$u(y)=A_1e^{s_1y}+A_2e^{-s_2y}+B_1e^{\lambda\mathrm{Pr}\,y}+B_2+R(a_2e^{q_1y}+a_6e^{-q_2y}+b_3+b_4e^{-\lambda\mathrm{Pr}\,y}$$
$$+b_5ye^{\lambda\mathrm{Pr}\,y}+b_6e^{2\lambda\mathrm{Pr}\,y}+b_7e^{3\lambda\mathrm{Pr}\,y}+b_8e^{4\lambda\mathrm{Pr}\,y}) \qquad (11)$$

## RESULTS AND DISCUSSION

This section illustrates the flow characteristics for the velocity, and temperature, then shows how the flow fields are influenced by the material parameters of the flow problems with the aid of graphs. Figure.2a,b delineate that velocity decreases with the increase of the magnetic parameter (M) due to suction ($\lambda$<0). From these Figures it is observed that the working fluid parameter (Pr) has no effect. Figure 3*a,b* illustrate the effect of magnetic parameter (M) on velocity when Pr=7.0. It is observed that increases magnetic parameter (M) decreases the values of velocity. Furthermore the suction/injection parameter has no effect see Figure 2*a* and *b* respectively. The effect of ambient parameter $(rt)$ on velocity is demonstrated in Figure 3 a, and b. It is observed that velocity increases with increase of ambient parameter (rt) due to injection ($\lambda$>0). In Figure 4a, b the effect of convective heat transfer parameter ($Bi_1$) is depicted. As $Bi_1$ increases the velocity increases due to suction ($\lambda$<0). Figure 5a, b highlight the effect of convective heat transfer parameter ($Bi_2$). From these Figures it is shown that the value of velocity decreases with increase in convective heat transfer parameter (rt) due to injection ($\lambda$>0). In Figure 7a, b it is indicated that as the ambient temperature parameter ($rt$) increases the temperature increases due to suction ($\lambda$<0). However, the values of temperature appears to be high in Figure 7b (Pr=7.0) in comparison with Figure 7a when Pr = 0.71.

Figure 7a,b report the behaviors of temperature field due to changing behaviors $Bi_1$ and $Bi_2$ at y=0 and y=1 respectively. Figure 8a, b shows that as *Biot* number ($Bi_1$) (convective heat transfer coefficient) increases, the temperature increases($\boldsymbol{\lambda = -5}$), but the reverse case is observed in Figure 8b in the case of air (Pr = 0.71) when $\boldsymbol{\lambda = 5}$ with little effect. Figure 9a,b reveal that as *Biot* number ($Bi_2$) (heat transfer coefficient) increases, the temperature decreases, but the values appears to be higher due to injection ($\boldsymbol{\lambda = +5}$) in comparison with suction ($\boldsymbol{\lambda = -5}$) see Figure 9a and 9b respectively.

**Velocity field**

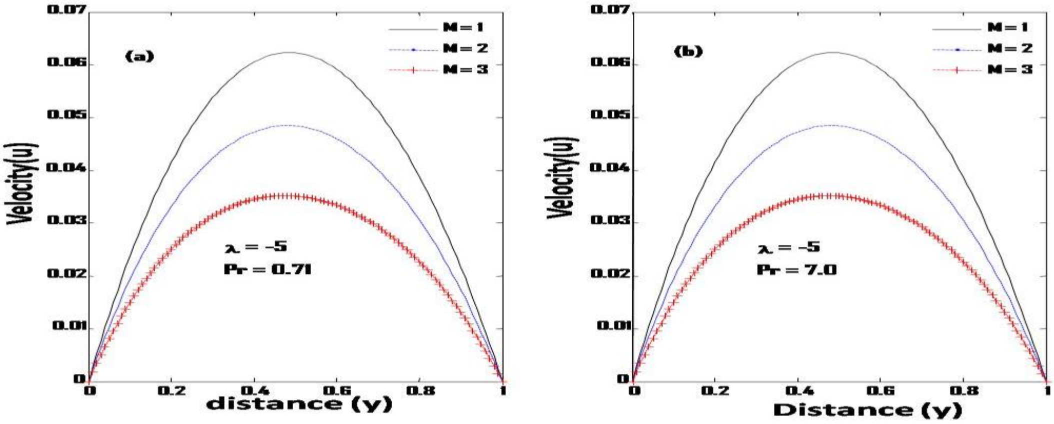

Figure 2a, b Effect of magnetic parameter (M) on velocity when: $Bi_1$ = 0.5, $Bi_2$ = 0.5, $C_T$ = 0.01, $rt$ = 0.01, R = 0.0001, Gr = 5

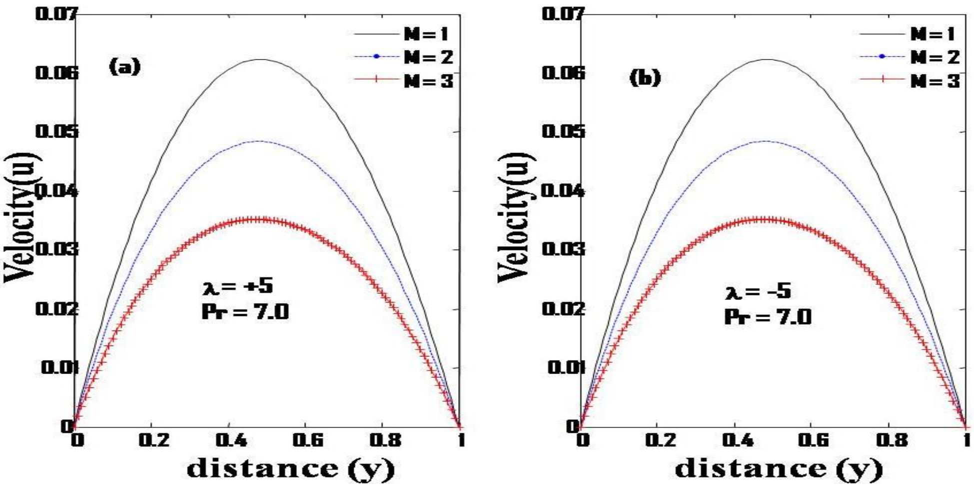

Figure 3a,b Effect of magnetic parameter (M) on velocity when: $Bi_1$ = 0.5, $Bi_2$ = 0.5, $C_T$ = 0.01, $rt$ = 0.01, R = 0.0001, Gr = 5

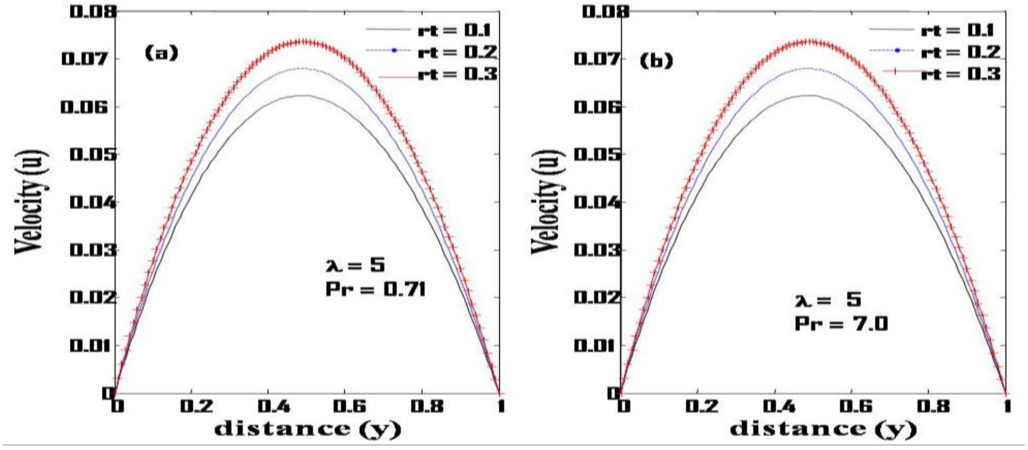

Figure 4a,b Effect of ambient parameter ( $rt$ ) on velocity when: $Bi_1$ = 0.5, $Bi_2$ = 0.5, $C_T$ = 0.01, Gr = 5, R = 0.0001, M =1.

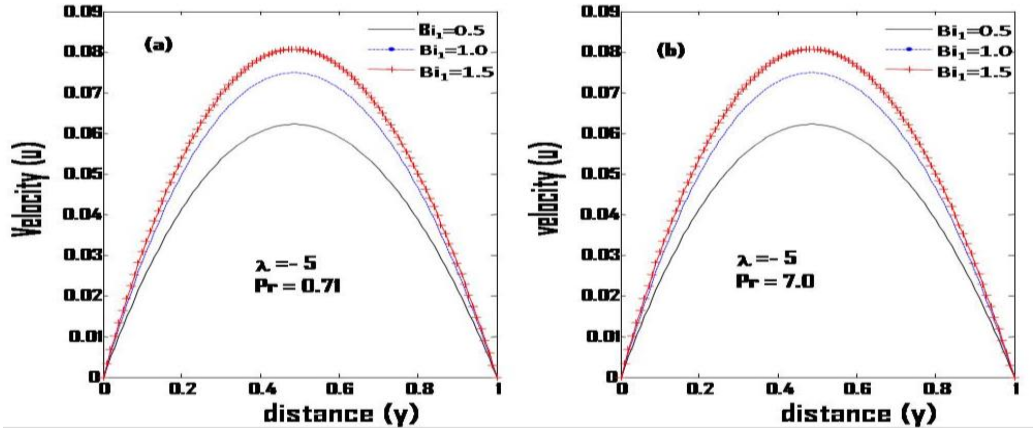

Figure 5a,b Effect of convective heat transfer parameter $(Bi_1)$ on velocity when $Bi_2$ = 0.5, $C_T$ = 0.01, Gr = 5, R = 0.0001, M =1.

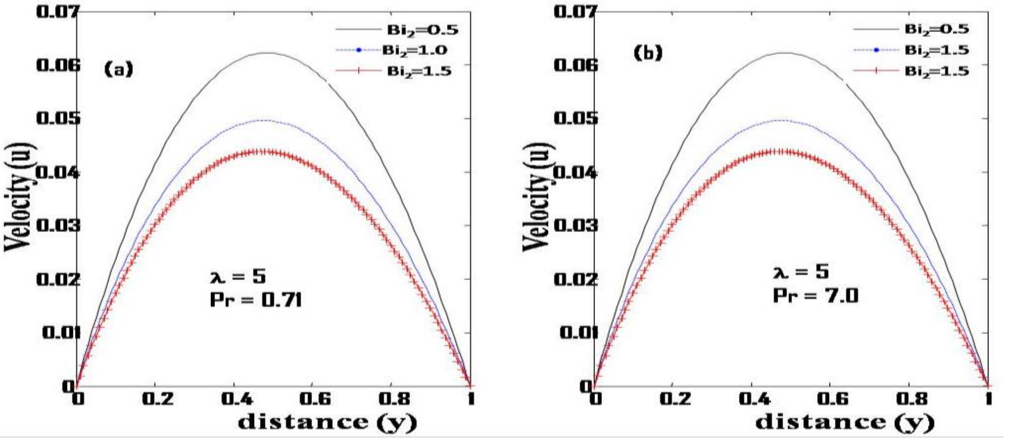

Figure 6a,b Effect of convective heat transfer parameter $(Bi_2)$ on velocity when: $Bi_1$ = 0.5, $C_T$ =0.01, Gr = 5, R = 0.0001, M =1.

**Temperature field**

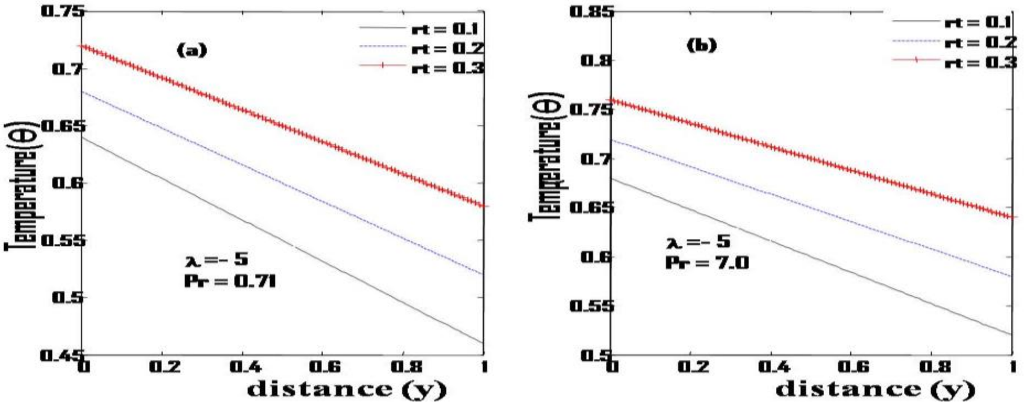

Figure 7a,b: Effect of ambient temperature parameter $(rt)$ on temperature when $Bi_1$ = 0.5, $Bi_2$ = 0.5, $C_T$ = 0.01, $rt$ = 0.01, R = 0.0001, M= 1

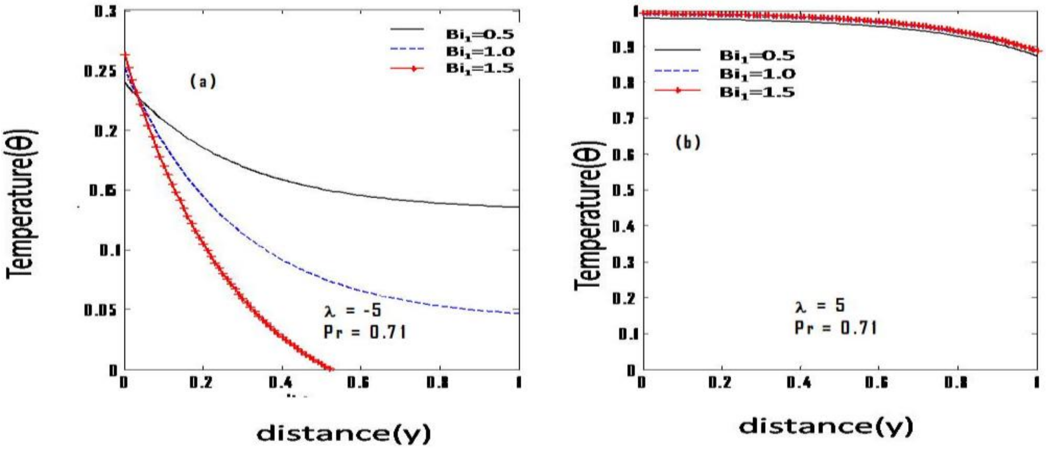

Figure 8a,b: Effect of convective heat transfer parameter ( $Bi_1$ ) on temperature when $Bi_2$ = 0.5, $\lambda = -5$, Pr = 0.71, $C_T$ = 0.01, $rt$ = 0.01, R = 0.0001, M = 1, Gr = 5

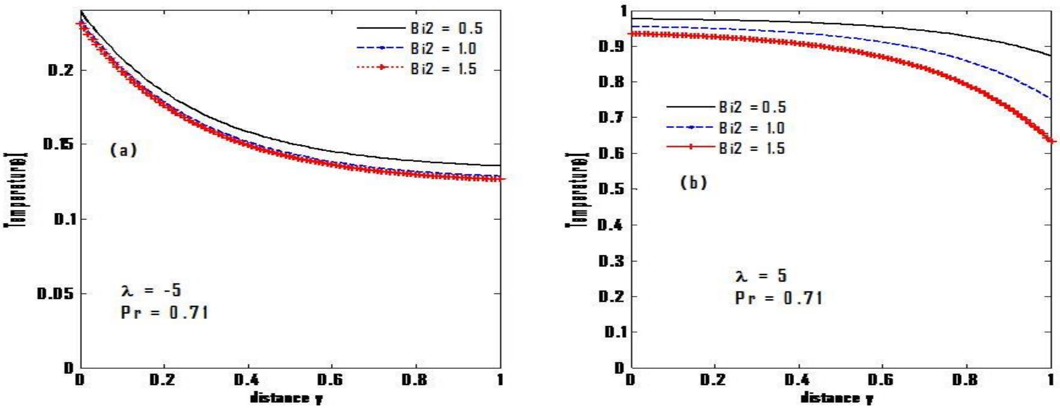

Figure 9a,b Effect of $Biot$ Number ($Bi_2$) on temperature when $Bi_1$ = 0.5,$C_T$ = 0.01, $rt$ = 0.01, R = 0.0001, M = 1, Gr = 5

## CONCLUSION

The problem of steady state convective flow of viscous fluid in a vertical channel with convective boundary condition has been investigated. The governing equations are solved by using perturbation method. The expression of velocity and temperature has been presented. The computations show that, flow formation is strongly depended on the controlling parameter within the flow formation. From the result the following conclusions were made;

i. Velocity decreases with increasing M, $rt$ , $Bi_1$, $Bi_2$ , and when $C_T = 0.001$.
ii. Velocity increases with an increase in $rt$ for both $C_T$=0.1 when $\lambda = \pm 5$
iii. Velocity decreases with increase in $rt$ $Bi_1$ and $Bi_2$ for $\lambda = \pm 5$·
iv. Temperature increases when $rt$ ($\lambda =$ -5) increase and also $Bi_2$ ($\lambda = 5$).
v. Temperature decrease when $Bi_1$ ($\lambda =$ -5) increases and also when $Bi_2$ ($\lambda = \pm 5$) increases.

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