*International Journal of Engineering and Mathematical Intelligence*
*Volume 4, Number 1; April 2017*
**ISSN: 2276-8815**

# Algorithm for Evaluation Changes in Linear Programming Using Kuhn-Tucker Condition

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## ABSTRACT

*In several problems of life confronting man, this study aims at finding out not only an optimal solution but also the fluctuating cost of raw materials, expected sales volumes, labour cost etc. The essence of the analysis is is to establish which constraint is binding or not binding. If the constraint is binding, therefore the variable is different from zero (0) else equate the variable to zero (0) with respect to any primal constraint for which equality seizes to hold at optimal solution, will have the value of the corresponding dual variable zero (0), but any constraint in the primal problem for which equality holds at optimal solution will have its corresponding dual variable different from zero. When changes are made in linear programming model, we sought for an optimal solution using an algorithm to determine the final value of the problem involving Kuhn-Tucker (K-T) conditions for a basic feasible solution which exist, a flowchart of the algorithm is presented in this work to enhance the understanding of the application.*
***Keywords:*** *Algorithm, changes, linear programming, Kuhn-Tucker Condition*

## INTRODUCTION

Many parameters in system dynamic models represent qualities that are very difficult or even impossible to measure with a great deal of accuracy in reality. Some parameter values change in the real world and these analyses allow one to determine what level of accuracy is necessary for a parameter to make the model sufficiently useful and valid. For every maximization linear programming problem, there is a corresponding maximization of profit which equivalent to the minimization of the cost of operating the business. These pairs of closely related problems are called dual linear programming problems. The first problem for which dual is sought is referred to as the primal problem, which is the Lenear Programming (LP) model developed for a particular situation. The other related one is the dual one which is a closely related Mathematical problem that can be derived directly from the primal problem. The rules for formulating the symmetric or nonsystematic dual LP problem from the primal LP problem and otherwise are contained in Ekoko (1999). Thus, in a Lenear Programming model in which the number of variables is considerably smaller than the number of constraints, computational savings

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may be realized by solving the dual from which the primal solution is automatically determined. As a result, the objective value may continue to increase (maximization case) indefinitely. In this case, both the solution space and optimum objective value are unbounded (Taha, 2006). The practical application of linear programming can be hampered at times by imperfect knowledge of the necessary data or by a complete lack of data. Ekoko (2004) proposes an algorithm for determining the final maximum of the $\theta$ considered to be a linear function of a parameter.

## THE LINEAR PROGRAMMING PROBLEM

Maximize Z =

Subject to

$$\sum_{j=1}^{n} a_{ij} X_j \le b_i, i = 1,2,....m \quad and \quad X_j \ge 0, j = 1,2,.........n,$$

where $C_j$'s are the cost coefficients, $b_i$ are right-hand side (R.H.S) values which are the technological constraints coefficients. Changes are most commonly investigated in the objective functions and in the right—hand side values $b_1$, $b_2$ …….$b_m$. These may correspond to changes in the availability of raw materials or changes in the quality and quantity of goods produced. The detection of these outcomes is by critical analysis which is mainly on the primal-dual properties, Forrester (2001). Researchers have been in search of computational algorithms for determining the maximum value of the parameter earlier stated, they have been in critical need and one is presented in this work.

When graph is used to solve linear programming problem, the use of slack and surplus variables do not arise. However, there is a theory called Kuhn-Tucker (K-T) conditions which states that any primal constraint for which equality does not hold at optimal solution, will have the value of its corresponding dual variable zero. That is any constraint that is **NOT BINDING** will have dual variable equal to zero. But a constraint in the primal problem for which equality holds at optimal solution will have its corresponding dual variable different from zero. Note that this method is used in the absence of simplex tables. An algorithm that could observe continuous changes in the cost coefficient of a linear programming problem is proposed using Kuhn-Tucker (K-T) conditions. The algorithm is intensive and a formal presentation of the steps with some explanation is given below.

**Step 1**: Involves expressing the primal problem

Maximize Z = $\sum_{j=1}^{n} C_j X_j$

S.T

$$\sum_{j=1}^{n} a_{ij} X_j \leq i = 1,2, X_j \geq 0,1,2$$

in its computational form (i.e thus putting the constraint in equation form without slack or surplus variable and initializing the optimum of the primal and the optimum the dual problem)

**Step 2:** Compute for the variables using the

$$\sum_{j=1}^{n} a_{ij} X_j \leq b_i, i = 1,2,.......m \text{ and } X_j \geq 0, j = 1,2.....m$$

**Step 3:**
Determine the optimum solution of the original primal problem using the equation of the objective function.

Maximizes $Z = \sum_{j=1}^{n} C_j X_{j,} \; j = 1,2, \ldots \text{n}$

**Step 4:** Verify if the optimum solution is equal to the optimum dual solution. If yes go to step 9 else continue

**Step 5:** Convert to the corresponding dual problem.

Maximizing $Y_o = \sum_{j=1}^{n} b_j y_j$

$$\sum_{j=1}^{m} a_{ij} Y_j \geq C_j, j = 1,2, \ldots \text{m}$$

$$y_j \geq 0, j = 1,2,...m$$

**Step 6:** If the constraint is binding therefore, the variable is different from zero (0) else equate the variable to zero (0) go to step 9.

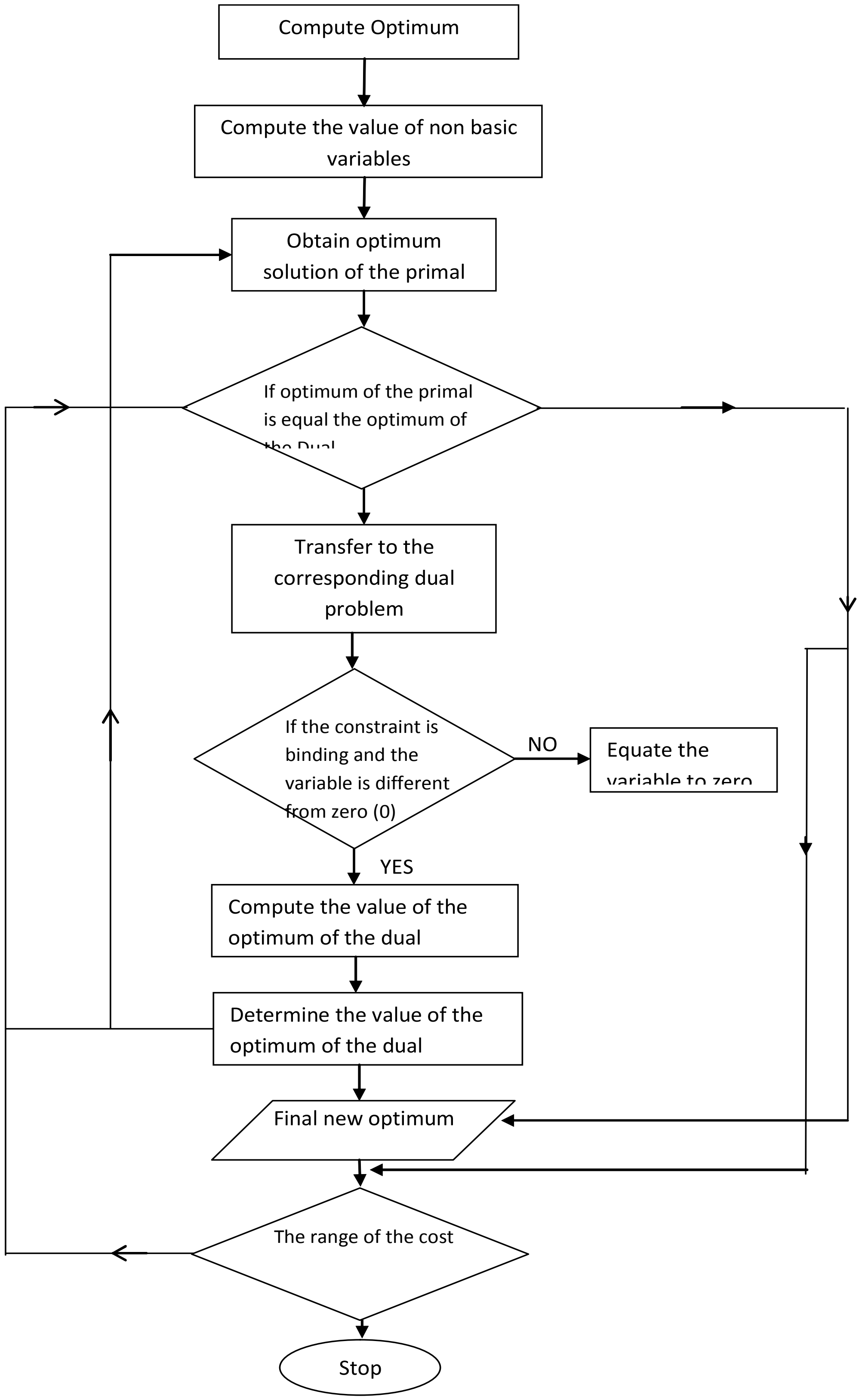
**Step 7:** Compute the value of the variable of the dual problem.

**Step 8**: Determine the value of the optimum of the dual solution and Go to step 4.

$$y_o = \sum_{j=1}^{m} b_j y_j, \quad j = 1,2....m$$

**Step 9:** Display final solution.

**Step 10:** If there is changes in the cost coefficient i.e thus cost coefficient are open and termination point take $C_{max}$ as final, else Go to step 3.


Compute Optimum
Compute the value of non basic variables
Obtain optimum solution of the primal
If optimum of the primal is equal the optimum of the Dual
Transfer to the corresponding dual problem
If the constraint is binding and the variable is different from zero (0)
NO
Equate the variable to zero
YES
Compute the value of the optimum of the dual
Determine the value of the optimum of the dual
Final new optimum
The range of the cost
Stop

A proof to help us have a better understanding of the steps in the algorithm

$$X_o \le \sum_{j=1}^{n} C_j X_j \left[ \sum_{i=1}^{m} a_{ij}, Y_i \right] x_j$$

$$X_o \le \sum_{j=1}^{n} \left[ \sum_{i=1}^{m} a_{ij}, X_j \right] y_j$$

$$X_o \le \sum_{i=1}^{m} b_i Y_i = y_o$$

Hence, the primal problem

Maximize $X_o = \sum_{j=1}^{n} C_j X_j$

Subject to

$$X_o = \sum_{j=1}^{n} a_{ij} X_j \le b_i \ I = 1,2,..........m$$

$$X_j \ge 0, \ j = 1,2,...........n$$

$$X_o = \sum_{j=1}^{n} C_j X_j$$

Can be transformed to its associated dual in the form

Maximize $Y_o = \sum_{j=1}^{m} b_j Y_j$

Subject to

$$\sum_{j=1}^{n} a_{ij} Y_i \ge C_j \ J = 1,2,..........n$$

$$Y_i \ge 0, i = 1,2,...........m$$

## CONCLUSION

In several problems of life confronting man, to find out not only an optimal solution but also the fluctuating cost of raw materials, expected sales volumes, labour cost etc is obvious. What we want to know from the analysis is which constraint is binding or not binding in step 6 with respect to any primal constraint for which equality seizes to hold at optimal solution, will have the value of the corresponding dual variable zero (0), but any constraint in the primal problem for which equality holds at optimal solution will have its corresponding dual variable different from zero.

Then we can concentrate on getting accurate data for those items or at least running several with various values of the crucial data in place of generate an idea of the range of possible outcomes. Since we live in a dynamic world where constraints are inevitable the study of the algorithm and flowchart on the solution due to changes in the data of a problem is very useful to observe continuous change in the cost coefficient of a linear programming problem using Kuhn-Tucker (K-T) conditions

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