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Mathematical Modelling Approach For Sensitivity and Stability Analyses of Cholera Disease in Aquatic Habitat

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ABSTRACT

A research is conducted on mathematical modeling approach for sensitivity and stability analyses of cholera disease in aquatic habitat. A deterministic mathematical model is formulated in the analyses of the degree of sensitivity and stability of the dynamical system which aid cholera transmission, spread and control. A numerical approach was adopted using the non-linear (autonomous 1st order) ordinary differential equations (ODE45 numerical scheme) to tackle the problem of sensitivity and stability. Results of sensitivity and stability analyses have significant epidemiological importance in Cholera control. Sensitivity indices of the basic reproduction number are derived, existence and stability of the model steady state based on threshold value were shown. The study further shows that long-term precise predictions of the concentration of infected cells during cholera could be difficult until these key parameters are correctly determined. These results are vital in the ongoing cholera vaccine development. An important parameter to cholera transmission is the contact between susceptible and infected persons, while a crucial parameter to cholera control is the rate of cholera awareness.

Keywords: *Sensitivity, stability, dynamical system, cholera transmission, numerical scheme*

1.0 INTRODUCTION

Living and non-living organisms co-exist and interact with the environment, and in the process get infected or polluted with the activities on the environment which in turn generate into disease and sickness. The cholera epidemic is a fatal water-borne disease causing diarrhea, dehydration, and vomiting in an individual (WHO, 2019).

Cholera is transmitted through ingesting contaminated feaces and touching vomit and corpse killed by the bacterium without using protective devices (Panja, 2019). The disease has incubation period of less than 24 hours to 5 days and the infection is often asymptomatic. Not greater than 25% of the infected persons are asymptomatic and of these 10 - 20% of the infected persons show severe symptoms. A constant loss of body fluids leads to dehydration, and rejecting treatment as the incident occurs, hastens the death of the infected person within a few hours (Mosler & Kessely, 2015).

Sensitivity and stability analyses in mathematical biology with respect to cholera diseases tell us how important each parameter is to the disease (cholera) transmission and is used to access how sensitive a model is to variation in the value of the parameters of the model and the changes in the structure of the model (Numfor, 2010 and Rodrigues et al., 2013). It also describes which parameters are vital in contributing to the prediction imprecision (that is how do changes in the values of the input parameter alter the value of the outcome variable) (Blower & Dowlatabadi, 1994). Above all, it is used to uncover parameters with high impact on the basic reproduction number R_o, so that it is directly targeted by intervention strategies. Over the past few years, sensitivity and stability analyses of cholera models have received attention of modelers and thereby become a subject of intense study. Many models have been developed to access sensitivity and stability of each factor driving the disease to ascertain the major factors to be targeted by the relevant interventions to eradicate the disease which had been in existence for over 200 years (Neilean *et al.*, 2010). Sensitivity and stability analyses of cholera tells us how diseases are transmitted which is greatly through aquatic habitat and the mathematical models used to study or uncover the parameter with high input on the basic reproduction number so that that it will directly target the intervention strategies. The epidemic outbreaks in susceptible populations, many of these factors are ignored; models assume a single infecting strain, an entirely susceptible population, and a short time scale for the epidemic can be neglected.

Sensitivity analysis can determine how variability of the input causes variability in the output. That is to quantify the ratio of output perturbations with respect to the input perturbations. The sensitivity and stability of cholera in aquatic habitat is carried out by dividing the population denoted by N(t) according to the infection status into S(t) – susceptible, I(t) – infected, R(t) – recovered and B(t) is the amount of concentration of vibrio cholarae in aquatic habitat at time t. And the model parameters are π, human recruitment rate, ξ, the rate of human contribution to the population of Vibrio cholera, β_1, rate of human exposure to contaminated water, δ, natural death rate of vibrio cholera, μ, death rate unrelated to cholera, μ_c, death rate due to cholera, β_2, rate of contact between susceptible and infected persons, θ, rate of cholera awareness, ω, rate of sanitation, υ, rate of vaccination, ρ, rate of cholera treatment and N, pathogen concentration that yields 50% catching cholera.

A mathematical model or frame work is formulated to analyze the degree of sensitivity and stability of some factors that aid cholera transmission and possible management. The disease free equilibrium point and local stability of the disease free and endemic equilibra of the model will be obtained in analysis. The reproduction number Ro with interactions are stated and the method of normalized forward sensitivity index is employed to determine the numerical value of the key model parameters with respect to the effective reproduction number to determine their relative importance of cholera transmission and its management.

The aim of this study is to carry out sensitivity and stability analyses of cholera infection using mathematical model when disease occurs in a population involving susceptible, infected and recovered persons in an aquatic habitat, and the behavior of control variables of the cholera infection due to the rate of human exposure to contaminated water. It will help in identifying influential model parameters and optimizing model to quantify the effects of sensitivity and stability on input parameters, and the subsequent effect on the models output.

2.0 Theoretical Assumptions

In disease model formulation, we make simplifications and assumptions on the model itself and on the parameters that represent the different transition and interaction in the model in question. Owing to the changes on parameter values, it is important to correctly understand the possible effects of such parameter values to the expected model output (Stockholm, 2006). Sensitivity and stability in the set of parameter values create variability in the models predictive capabilities. The less the number of changes in parameters in the model, the less the significance of variability introduced into a model, and vice-versa (Gomero, 2012).

Edward & Nyerere (2015) made a mathematical model that entailed some vital dynamics of cholera transmission with public health educational campaigns, vaccination, sanitation and treatment as control strategies in limiting the disease. Sensitivity analysis was carried out by them in the basic reproduction number with all control strategies and was discovered that the most sensitive parameters are educational campaigns, therapeutic treatment, and effective contact between the

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susceptible and infected persons, bacteria carrying capacity and recruitment rate. And they concluded that any strategy aimed at eliminating cholera should target these parameters. Mondal & Kar (2013) showed a waterborne disease epidemic model amenable to cholera dynamics including multiple transmission namely, water-to-person and person-to-person transmission. Their study of the sensitivity analysis of the system in reference to some crucial parameters and discovering of the number of infected persons and pathogen concentrations were directly proportional to the two type disease transmission rate. It was also discovered that if person-to-person contact was not applied, then the disease may be transmitted through the contaminated reservoir and within a very small time it might spread into the entire population. Kadeleka (2011) carried out a study by formulating a basic mathematical model to access the effects of nutrition and treatment of cholera dynamics, and the computational results indicated that the cholera epidemic can be controlled when intervention, nutrition and treatment are introduced.

The disease model provides a mathematical representation of the dynamic transmission cycle, involving interactions between infected and susceptible hosts that are generally expressed as a set of coupled ordinary differential equations (ODEs) (Keeling, 2007). The model outputs (the ODE solutions over a simulation interval), provides a dynamic representation of the transmission process. Fraser (2008) said that model outputs often have complex, non-linear relationship with model parameter, values and inappropriate choices of parameter values can lead to bias in model outputs (Ecohard 2010). Sensitivity analysis characterizes the response of model outputs to parameter variations, (Tarantola 2008). Sensitivity and stability can affect the reliability of the results at every stage of computations; they may grow or shrink as the solution of the model evolves.

3.0 MATERIALS AND METHODS

3.1 Model Formation

The interaction between the population of susceptible persons, the infected, the recovered persons and the vibrio cholerae concentration in the aquatic habitat which are time dependent is being studied in formulating the system of non-linear ordinary differential equations. This mathematical formulation describes an aspect of vibrio cholerae infection which defines a set of deterministic values with application in health informatics modeling.

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3.2 Mathematical Formation

The model used in this research work is a set of deterministic system of time non-linear first order differential equation proposed by Wang and Mondak (2011) which describes the transmission of the vibrio cholerae infection.

3.3 Mathematical Modeling

This refers to the process of describing a system by means of mathematical concepts and language. In other words, mathematical modeling is a process of encoding and decoding reality in which a natural phenomenon is reduced to a formal numerical expression by a causal process.

3.4 Computational Approach

This is the study that straddles the artificial intelligence and psychology divide. It is concerned with developing computer models of human cognitive process and is based on a mental, brain and computer program analogy. Computational models are Mathematical models that employ a computer simulation to quantitatively examine the behaviour of a complicated system. A computational model can be used to forecast the behviour of a system under various conditions, which is useful in situation where straight forward analytical solution are not readily accessible.

Table 3.1: The definition for the model parameter, their values, units and source

Parameter	**Symbol**	**Values**	**Units**	**Sources**
Human recruitment rate	π	10	day^{-1}	Kadeleka, 2011
Rate of Human contribution to the population of Vibrio Cholerae	ξ	10	cells/μl/day	Isere et al, 2014
Rate of human exposure to contaminated water	β_1	0.075	day^{-1}	Wang, 2011
Pathogen concentration that yields 50% chance of catching cholera	N	10^5	cells/μl	Edward & Nyerere, 2015
Natural death rate for Vibrio Cholerae	ν	0.4	day^{-1}	Isere et al, 2014
Death rate unrelated to Cholera	μ	0.02	day^{-1}	Kadeleka, 2011
Human death rate due to cholera	μ_c	0.015	day^{-1}	Kadeleka, 2011
Rate of contact between susceptible and infected persons	β_2	0.00011	day^{-1}	Wang & Modnak, 2011
Rate of Cholera awareness	θ	0.6	Dimensionless	Assumed
Rate of Vaccination	σ	0.2	Dimensionless	Assumed
Rate of Sanitation	ω	0.5	Dimensionless	Ochoche, 2013
Rate of Cholera treatment	ρ	0.98	day^{-1}	Kadeleka, 2011

***Source*:** Wang X. and Wang J. (2014)

3.5 Method of Solution

$$\frac{ds}{dt} = \pi - \mu s - (1-\theta)\frac{\beta_1 BS}{B+N} - (1-\theta)\beta_2 IS + \sigma R - \upsilon S \quad (3.1)$$

$$\frac{dI}{dt} = (1-\theta)\beta_1 BS + (1-\theta)\beta_2 IS - (\mu + \mu c + \rho)I \quad (3.2)$$

$$\frac{dR}{dt} = \sigma s - \mu R - \sigma R + \rho I \quad (3.3)$$

$$\frac{dB}{dt} = (1-\theta)\xi I - (\sigma + \omega)B \quad (3.4)$$

Determination of the Steady State Solution

At steady state,

$$\frac{ds}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = \frac{dB}{dt} = 0 \quad (3.5)$$

From (3.1), $\frac{dS}{dt} = 0$. Using (3.5) in (3.1) – (3.4) yields (3.6), (3.7), (3.8) and (3.9) respectively

$$\Rightarrow \pi - \mu S - (1-\theta)\frac{\beta_1 BS}{B+N} - (1-\theta)\beta_2 IS + \sigma R - \upsilon S = 0$$

$$\Rightarrow \pi(B+N) - \mu S(B+N) - (1-\theta)\beta_1 BS - (1-\theta)(B+N)\beta_2 IS + \sigma R(B+N) - \upsilon S(B+N) = 0$$

$$\Rightarrow \pi B + \pi N - \mu BS - \mu NS - (1-\theta)\beta_1 BS - (1-\theta)\beta_2 IBS - (1-\theta)\boldsymbol{N}\beta_2 IS + \sigma BR + \sigma NR - \upsilon BS - N\upsilon S = 0$$

$$\Rightarrow \pi B - \mu BS - \mu NS - (1-\theta)\beta_1 BS - (1-\theta)\beta_2 IBS - (1-\theta)\boldsymbol{N}\beta_2 IS + \sigma BR + \sigma NR - \upsilon BS - N\upsilon S = -\pi N \quad (3.6)$$

From (3.2);

$$\frac{dI}{dt} = 0,$$

$$\Rightarrow (1-\theta)\beta_1 BS + (1-\theta)\beta_2 IS - (\mu + \mu c + \rho)I = 0 \quad (3.7)$$

From (3.3)

$$\frac{dR}{dt} = 0,$$

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$\Rightarrow \sigma S - \mu R - \sigma R + \rho I = 0$ (3.8)

And from (3.4)
$\frac{dB}{dt} = 0,$
$\Rightarrow (1-\theta)\xi I - (\sigma+\omega)B = 0$
$\Rightarrow \mathrm{I} = \frac{(\sigma+\omega)B}{(1-\theta)\xi} = 0$ (3.9)

Putting (3.9) in (3.8); we have
$\sigma S - (\mu+\sigma)R + \rho\frac{(\sigma+\omega)B}{(1-\theta)\xi} = 0$
$\Rightarrow \sigma\xi(1-\theta)S - (1-\theta)(\mu+\sigma)\xi R + \rho(\sigma+\omega)B = 0$ (3.10)

Putting (3.9) in (3.7), then
$(1-\theta)\beta_1 BS + \frac{(\sigma+\omega)\beta_2 BS}{\xi} - \frac{(\sigma+\omega)(\mu+\mu c+\rho)B}{(1-\theta)\xi} = 0$

$\Rightarrow (1-\theta)^2\xi\beta_1 BS + (1-\theta)(\sigma+\omega)\beta_2 BS - (\sigma+\omega)(\mu+\mu c+\rho)B = 0$

$\Rightarrow \mathrm{B}[(1-\theta)^2\xi\beta_1 S + (1-\theta)(\sigma+\omega)\beta_2 S - (\sigma+\omega)(\mu+\mu c+\rho)] = 0$

$\Rightarrow B = 0$ or

$(1-\theta)^2\xi\beta_1 BS + (1-\theta)((\sigma+\omega)\beta_2 BS - (\sigma+\omega)(\mu+\mu c+\rho)B = 0$

$\Rightarrow [(1-\theta)^2\xi\beta_1 + (1-\theta)(\sigma+\omega)\beta_2]S = (\sigma+\omega)(\mu+\mu c+\rho)]$. [$B^*$ is an assumption]

$S = \frac{(\sigma+\omega)(\mu+\mu c+\rho)B}{(1-\theta)^2\xi\beta_1 + (1-\theta)(\sigma+\omega)\beta_2}$ S^* (3.11)

But by putting B=0 in (3.10), it becomes
$\sigma\xi(1-\theta)S - (1-\theta)(\mu+\sigma)\xi R = 0 \Rightarrow \sigma S - (\mu+\sigma)R = 0$
$\Rightarrow R = \frac{\sigma S}{\mu\sigma}$ (3.12)

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Putting B = 0 and I = 0 in (3.6) we have

$-\mu NS + \sigma NR - N\nu S = -\pi N \Rightarrow \mu S - \sigma R + \nu S = \pi$

$\Rightarrow (\mu + \nu)S - \sigma R = \pi$ (3.13)

Putting (3.12) in (3.13), then

$$(\mu + \nu)S - \frac{\sigma^2 R}{\mu + \sigma} = \pi$$

$\Rightarrow (\mu + \sigma)(\mu + \nu)S - \sigma^2 S = \pi(\mu + \sigma)$

$\Rightarrow [(\mu + \sigma)\mu + \nu) - \sigma^2]S = \pi(\mu + \sigma).$

$S = \frac{\pi(\mu+\sigma)}{(\mu+\sigma)(\mu+\nu)-\sigma^2} = S\text{\textasciicircum}$ ($S^\wedge$ is an assumption) (3.14)

Putting (3.14) in (3.12); then

$R = \frac{\pi\sigma(\mu+\sigma)}{(\mu+\sigma)^2(\mu+\nu)-\sigma^2(\mu+\sigma)}$

$R = \frac{\pi\sigma}{(\mu+\sigma)(\mu+\nu)-\sigma^2} = R^\wedge$ ($R^\wedge$ is an assumption) (3.15)

4.0 RESULTS AND DISCUSSION

The sensitivity analysis (test) of cholera in aquatic habitat using the mathematical model:

$\frac{ds}{dt} = \pi - \mu S + \sigma R - \nu S$

Table 4.1: Sensitivity Analysis $R = 40.6$, $S = 5.0$, $\gamma = 0.02$, π – varies

π	μ	$\frac{ds}{dt} = \pi - \mu S + \sigma R - \nu S$
10	0.020	15.90
11	0.018	16.91
12	0.016	17.92
13	0.014	18.93
14	0.012	19.94
15	0.010	20.95
16	0.008	21.96
17	0.006	22.97
18	0.004	23.98
19	0.002	24.99
20	0.001	25.995

Source: Umoh, E.S. (2022)

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In Table 4.1, for a varying value of π (human recruitment rate), 1.00 – 10.00 results in an increase in the value of the sensitivity of susceptible persons, S with μ ranging from 0.02 – 0.002, whereas fro table 4.1.4, a reduction in the value of π (10.00 – 1.00) and μ (0.02 – 0.002) indicates a decrease in the sensitivity of susceptible persons. The varying π, from 10 to 20, and varying μ from 0.020 to 0.002, the dependent variable increases from 15.90 to 25.995, equally predicting a decrease in the sensitivity of susceptible persons to cholera infection in that habitat. On the sensitivity of susceptible persons scenario, the increase in human recruitment rate, predict a decrease in the sensitivity of susceptible persons, which is consistent with the work of Nwagor and Ekaka-a (2017).

Table 4.2: Quantifying the effect of 10% variation of rate of human exposure to contaminated water on the dynamical system($\beta_1 = 0.075$)

Time (t)	β_1	*S*(t)	*I*(t)	*R(t)*	*B(t)*
0.00	0.0075	1000.000000000000	50.000000000000	20.000000000000	120.000000000000
0.05	0.0075	979.591987364246	81.182300010017	45.500872272003	134.832357682902
0.10	0.0075	960.024498818397	114.409772291615	73.148413974624	160.612234385295
0.15	0.0075	941.314692627377	152.267176422852	103.246551833984	197.727373661316
0.20	0.0075	923.479494323779	197.235773857825	136.320740289395	247.415720551232
0.25	0.0075	906.537839133412	251.789364962090	173.137068246353	311.816693264859
0.30	0.0075	890.517824728392	318.712935297366	214.690885173226	393.843572322504
0.35	0.0075	875458559092120	401.174003936125	262.248363971623	497.321181731835
0.40	0.0075	861.415805989765	502.971972257721	317.383049885002	627.055239804064
0.45	0.0075	848.464517006677	628.647498484801	382.039001757182	789.041391871032
0.50	0.0075	836.705076565613	783.779753605760	458.608920768520	990.681518202128
0.55	0.0075	826.266767344155	975.179478878350	550.032103943846	1241.108843301104
0.60	0.0075	817.315692852690	1211.339808401815	659.928660310351	1551.587823269608
0.65	0.0075	810.059331135194	1502.807480240954	792.757378258524	1936.040357762632
0.70	0.0075	804.756634565558	1862.954082082231	954.045536990950	2411.758504449836
0.75	0.0075	801.723504276388	2308.732254439054	1150.656879514565	3000.302721334482
0.80	0.0075	801.344639168330	2862.101997841409	1391.203215812687	3728.823310399562
0.85	0.0075	804.078747657475	3551.634584045606	1686.527131260514	4631.688463938120
0.90	0.0075	810.469156535430	4415.318514607173	2050.482021326027	5753.068372252369
0.95	0.0075	821.144526934345	5504.061264432333	2500.854823502280	7150.065517697770
1.00	0.0075	836.817790043535	6887.470724293232	3060.905766898180	8897.946665264065

Table 4.2 shows the contribution of contact between susceptible and infected persons on the dynamic system. The smaller value of the contact rate between the susceptible and infected persons (β_2) (ranging from 0.000011 to 0.000099) yield or predict a relatively high value of λ. Whereas, an increased value of β_2, predicted smaller values of λ resulting to unstable steady state solution of the dynamical system. By quantifying 10% variation of human exposure to contaminated water, the susceptible

persons' value decreases, unlike the infected, recovered and total concentration of vibrio cholera which shows a remarkable increase in population. On the contribution of contact between susceptible and infected persons on the dynamical system, and variation of the rate of human exposure to contaminated water, predict a predominantly unstable steady state solution on the dynamical system.

Table 4.3: Quantifying the effect of 20% variation of rate of human exposure to contaminated water on the dynamical system$(\beta_1 = 0.075)$

Time(t)	β_1	*S*(t)	*I*(t)	*R(t)*	*B(t)*
0.00	0.015	1000.0000000000	50.0000000000	20.0000000000	120.0000000000
0.05	0.015	979.5247717019	118.3546950346	47.2743789577	1419.132023774
0.10	0.015	959.7925501900	198.2540373023	80.6351277163	1898.958408994
0.15	0.015	940.8582508693	303.1484535409	121.7374979612	2687.280383076
0.20	0.015	922.7673018559	448.2448515860	173.5651582395	3881.687355685
0.25	0.015	905.5777276517	653.0410384695	240.7084027867	5637.222849946
0.30	0.015	889.3760734270	943.6013734849	329.8291484704	8181.629214136
0.35	0.015	874.3014351320	1356.1569346666	450.4690786378	11842.192908184
0.40	0.015	860.5660767177	1940.8328239316	616.0654332684	17080.620211227
0.45	0.015	848.4927651619	2768.4831028998	845.6481291009	24550.693581087
0.50	0.015	838.5475934918	3938.4155158369	1165.8996927391	35169.514878353
0.55	0.015	831.3932896742	5593.3457331281	1614.7037992287	50239.435974208
0.60	0.015	827.9283737027	7937.7509954822	2245.5036780212	71599.781227508
0.65	0.015	829.3174922622	11274.9253039148	3135.3157907686	101906.698532478
0.70	0.015	836.9748401065	16056.2475884589	4394.8319175857	144988.334175468
0.75	0.015	852.3711124423	22982.1126689830	6188.4988237785	206559.501230407
0.80	0.015	876.7187351385	33141.3360043827	8760.3714659819	295139.768231085
0.85	0.015	909.9695495064	48283.1789899114	12487.6710772916	423984.738507369
0.90	0.015	949.8563472918	71164.1927868801	17949.2075529415	613525.382608508
0.95	0.015	990.1980373529	106122.4718418230	26056.5926781758	896081.786101024
1.00	0.015	1022.1379076303	159595.5334627266	38214.2903453303	1321500.904060970

Table 4.3 shows that a 20% variation of the rate of human exposure to contaminated water at a constant value of $\beta_1 = 0.015$ yielded a relative increase in the value of susceptible persons, infected recovered and the total cholera population with fluctuation in the susceptible persons. By quantifying 20% variation of human exposure to contaminated water, the susceptible persons' value decreases, unlike the infected, recovered and total concentration of vibrio cholera which shows a remarkable increase in population.

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Table 4.4: Quantifying the effect of 10% variation of rate of contact between susceptible and infected persons on the dynamical system$(\beta_2 = 0.00011)$

Time (t)	β_2	*S*(t)	*I*(t)	*R(t)*	*B(t)*
0.00	0.000011	1000.00000000	50.00000000	20.00000000	120.00000000
0.05	0.000011	989.79913392	234.17803476	36.51683663	142.27640358
0.10	0.000011	979.87616098	466.76102782	62.69250908	203.48864085
0.15	0.000011	970.32353896	807.99472548	102.19301397	317.08217256
0.20	0.000011	961.27106001	1341.68471417	162.08443747	509.71696246
0.25	0.000011	952.91774468	2194.48072004	254.39285814	827.12967436
0.30	0.000011	945.58421523	3567.56844539	398.86288647	1344.55391231
0.35	0.000011	939.78498539	5781.69463603	627.47961125	2183.98442080
0.40	0.000011	936.35452330	9358.00828980	992.08193591	3543.17115363
0.45	0.000011	936.63316235	15145.57836600	1576.49405324	5741.99392631
0.50	0.000011	942.78047840	24561.06049793	2518.00202900	9304.66721372
0.55	0.000011	958.25752688	40012.72500020	4042.46530177	15094.31919962
0.60	0.000011	988.58543503	65777.18767535	6533.41101042	24579.67400431
0.65	0.000011	1042.61199115	109822.50240127	10653.38135437	40302.72767256
0.70	0.000011	1134.02578860	188075.66655015	17627.96768232	66988.16995387
0.75	0.000011	1284.42613285	335106.83123722	29801.43986815	113699.88013467
0.80	0.000011	1521.72676010	632324.81765584	52216.76602222	200075.31939831
0.85	0.000011	1874.27372767	1291267.56594397	96343.10347066	370936.67223750
0.90	0.000011	2308.33492848	2873840.20395146	191092.14513541	739875.50533415
0.95	0.000011	2619.79730492	6814400.93741627	411071.60893039	1600566.50481003
1.00	0.000011	2638.66745859	16124877.23187691	934934.99752241	3655523.97738281

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Table 4.4 shows the behavior of the control variable of the cholera infection due to variation of the data set of human exposure to contaminated water, when β_1 is constant at 0.075. The value of the susceptible persons ranges from 50.0000 to 1900669211.471159. The recovered persons ranges from 20.0000 to 243802383.016390, whereas the population of cholera (bacteria) in the aquatic habitat which ranges from 120.0000 to 91378703080.07970. By quantifying 10% variation of rate of contact between susceptible and infected persons, and at constant β_2, the susceptible persons' value increases, likewise the infected, recovered and total concentration of vibrio cholera. The increase in the rate of contact between susceptible and infected persons predicts a reduction in the population of susceptible persons and increase in the concentration of cholera infection

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Table 4.5: Quantifying the effect of 20% variation of rate of contact between susceptible and infected persons on the dynamical system$(\beta_2 = 0.00011)$

Time (t)	β_2	*S*(t)	*I*(t)	*R(t)*	*B(t)*
0.00	0.000022	1000.000000000	50.000000000	20.000000000	120.000000000
0.05	0.000022	989.768606072	234.206697336	36.517275094	142.278658865
0.10	0.000022	979.771809519	466.854682455	62.695141214	203.502188609
0.15	0.000022	970.086948123	808.187887900	102.200864417	317.122811346
0.20	0.000022	960.814435560	1341.968856788	162.100668442	509.803309046
0.25	0.000022	952.103464909	2194.652741151	254.415531881	827.263554410
0.30	0.000022	944.193389609	3566.759722812	398.864500800	1344.640815305
0.35	0.000022	937.467604308	5777.022352568	627.349116771	2183.608047814
0.40	0.000022	932.548769609	9340.870223306	991.442427439	3540.874941192
0.45	0.000022	930.433174196	15091.607367805	1574.238048638	5733.460425427
0.50	0.000022	932.712632894	24403.257203989	2510.920474247	9277.299181748
0.55	0.000022	941.891490462	39568.891618056	4021.842209734	15013.869222148
0.60	0.000022	961.824215162	64552.151530075	6475.001889358	24350.577363844
0.65	0.000022	998.326894917	106447.256005927	10492.083288587	39668.513016143
0.70	0.000022	1059.357480321	178665.162412581	17178.749235978	65217.759857831
0.75	0.000022	1155.087573446	308013.149866675	28545.787343783	108746.260558930
0.80	0.000022	1291.647361810	550796.401747412	48563.282922771	185636.434778628
0.85	0.000022	1461.946621204	1028891.329600502	85195.880397459	326815.695009516
0.90	0.000022	1613.762393598	1996136.729533204	155283.595774585	597919.036288180
0.95	0.000022	1675.167348639	3927091.525916773	292837.525835975	1131455.203963971
1.00	0.000022	1667.479800141	7654207.146095336	562187.717389170	2177446.959462845

Table 4.5 shows a monotonically decreasing pattern in the values of $S(t)$ – susceptible persons per time from 1000.000000 to 801.3446. In contrast, the values of infected persons, recovered persons and the total cholera population increase monotonically from their critical values due to a small increase in the rate of human exposure to contaminated water. This is significant in mitigating cholera infection and relevant in public health policy initiatives. By quantifying 20% variation of rate of contact between susceptible and infected persons, and at constant β_2, the susceptible persons' value increases, likewise the infected, recovered and total concentration of vibrio cholera. On the behavior of the control variables of cholera infection, the population of infected persons, recovered persons, the total cholera concentration increases monotonically from their critical value due to a small increase in the rate of human exposure to contaminated water. This is significant in mitigating cholera infection and it is relevant in public health initiative.

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Table 4.6: Effect of human exposure to contaminated water on the stability of the dynamical system

β_1	λ_1	λ_2	λ_3	λ_4	TOS
0.075	-39812.71658669595	-17.41682184082	7.42003594031	-0.55361051806	Unstable
0.0075	-10.274244434883473	8.821119747921539	-0.554962766574937	-0.554962766574937	Unstable
0.015	-7.256844086446767	-7.256844086446767	8.091947754763949	-0.573257526961217	Unstable
0.0225	-23.095180665399170	-23.095180665399170	7.160009402147707	-0.551079284789429	Unstable
0.030	-153.8635556813403	-27.1159505968098	7.1799979842951	-0.5510483500691	Unstable
0.0375	-578.9367232325724	-22.8116228962738	7.2705089745925	-0.5516086485719	Unstable
0.0450	-1680.364280844664	-20.955689424133	7.337286181894	-0.552121208243	Unstable
0.0525	-4223.271160778493	-19.721078469626	7.379806616197	-0.552630083585	Unstable
0.060	-9590.259366973847	-18.785359328819	7.404475528078	-0.553073445315	Unstable
0.0675	-20153.50382187567	-18.04193055810	7.41655838946	-0.55329581072	Unstable
0.07125	-28523.37541459641	-17.71182692062	7.41923510726	-0.55353702236	Unstable
0.0735	-34896.45855069797	-17.53624561912	7.41990260902	-0.55349892068	Unstable
0.0825	-74762.01294088454	-16.88852503356	7.41729687167	-0.55381336388	Unstable
0.090	-134550.9262181156	-16.4259261863	7.4099976886	-0.5541029670	Unstable
0.0975	-233542.9296624187	-16.0209634156	7.3993965980	-0.5543457613	Unstable
0.105	-392881.0377219479	-15.6595619514	7.3862795538	-0.5546171875	Unstable
0.1125	-643102.2387031537	-15.3429756073	7.3716793771	-0.5546984094	Unstable
0.120	-1027592.758041501	-15.054745183	7.355684829	-0.554830182	Unstable
0.1275	-1607083.434771557	-14.790710709	7.338619109	-0.555003973	Unstable
0.135	-2465929.821303217	-14.549804605	7.320900144	-0.555152387	Unstable
0.1425	-3718034.128378208	-14.327417689	7.302627169	-0.555316507	Unstable
0.14625	-4538088.494748818	-14.222465972	7.293336797	-0.555402420	Unstable
0.1485	-5105291.435448762	-14.164305968	7.287981311	-0.555372186	Unstable

Table 4.7: Effect of contact between susceptible and infected persons on the dynamical system

β_2	λ_1	λ_2	λ_3	λ_4	TOS
0.000011	-279.9180826108713	-32.7046959188386	10.8053312218446	-0.2841587390234	Unstable
0.000022	-156.4572482671883	-27.4673811577599	9.1574477632315	-0.3337019730729	Unstable
0.000033	-104.5722007835395	-25.4336228543098	8.4447235686995	-0.3763937440429	Unstable
0.000044	-75.941480702552610	-24.645579250602612	8.059197715008937	-0.413200887920538	Unstable
0.000055	-57.359786113535542	-24.755609557963712	7.826607965349210	-0.445120650131634	Unstable
0.000066	-43.517016920323471	-25.998114822145990	7.676124148093995	-0.473151685242553	Unstable
0.000077	-30.193790730120305	-30.193790730120305	7.574706660603323	-0.497853989239400	Unstable
0.000088	-26.746292078205638	-26.746292078205638	7.503857182535888	-0.519787905813366	Unstable
0.000099	-24.073830501017767	-24.073830501017767	7.451977938562727	-0.539471509118388	Unstable
0.0001045	-22.942541473776398	-22.942541473776398	7.431358634429062	-0.548564146162581	Unstable
0.0001078	-22.313569469611540	-22.313569469611540	7.420839594284378	-0.553690917901242	Unstable
0.000121	-20.163273116596745	-20.163273116596745	7.384056538958928	-0.573173256132916	Unstable
0.000132	-18.701031746903503	-18.701031746903503	7.360383640682048	-0.587761920803750	Unstable
0.000143	-17.467408351561645	-17.467408351561645	7.340708064730648	-0.601109797949579	Unstable
0.000154	-16.413564350254426	-16.413564350254426	7.323711742241775	-0.613373071913307	Unstable
0.000165	-15.503493356122398	-15.503493356122398	7.308474253470843	-0.624682347831496	Unstable
0.000176	-14.710093239144708	-14.710093239144708	7.294347612642705	-0.635147789812080	Unstable
0.000187	-14.012591442106537	-14.012591442106537	7.280873943461584	-0.644863013632623	Unstable
0.000198	-13.394811322422317	-13.394811322422317	7.267729961416242	-0.653908103977543	Unstable
0.000209	-12.843976554422454	-12.843976554422454	7.254688742598846	-0.662351989202810	Unstable
0.0002145	-12.590397927028128	-12.590397927028128	7.248152339771663	-0.666368085965177	Unstable
0.0002178	-12.444596670422705	-12.444596670422705	7.244214630212560	-0.668716208672557	Unstable

CONCLUSION AND RECOMMENDATION

This work presents sensitivity and stability analyses, a mathematical model of infectious disease of cholera in aquatic ha using ODE45 numerical scheme. In the context of modeling of first order non-linear differential equation of a dynamical system involving cholera infection, and adopting a computational approach to investigate the sensitivity and stability analyses of the system under investigation which is another frontier of knowledge in mitigating cholera infection spread. The increase in the rate of contact between susceptible and infected persons predicts a reduction in the population of susceptible persons and increase in the concentration of cholera infection. Therefore, the numerical method utilized in this work can be extended in tackling the uncertainty on a dynamical system involving cholera infection.

REFERENCES

Blower, S.M. & Dowlatabadi, H. (1994). Sensitivity analysis of complex models of disease transmission: an HIV model as an example. *International Statistical Review, 62*(2), 229 –243.

Ecohard, R. (2010). Methodology of the sensitivity analysis used for modelling on infection disease. *Vaccine, 28*, 8132 –8140. doi:10.1016/j.vaccine.2010.09.099

Edward, S. & Nyerere, N. (2015).A mathematical model for the dynamics of cholera with control measures. *Application of Computational Mathematics*, *4(2),* 53 – 63.

Fraser, C. (2012). Mathematical models on infectious disease transmission. *National Reservoir Micro Biology, 6*, 477 –487.

Gomero, B. (2012). *Latin hypercube sampling and partial rank correlation coefficient analysis applied to an optimal control problem*. Master's Thesis, University of Tennessee. https://trace.tennessee.edu/utk_gradthes/1278

Isere A. O., Osemwenkhae J. E. & Okuonghae D. (2014). Optimal control model for the outbreak of cholera in Nigeria. *African Journal of Mathematics, Computer Science Research*, *7*(2), 24

Kadeleka, S. (2011). *Assessing the effects of nutrition and treatment in cholera dynamics: The case of Malawi.* M.Sc. Dissertation, University of Deres Salaam.

Keeling, M. (2007). *Modelling infectious disease in human and animal*. Princeton University Press.

Mondal, P. K. & Kar, J. K. (2013). Global dynamics of waterborne disease model with multiple transmission pathways .*Applications and Applied Mathematics, 8*(1), 75 – 98.

Mosler, J. & Kessely, H. (2015). Factors determining water treatment behaviour for the prevention of cholera treatment in Chad. *The American Journal of Tropical Medicine and Hygiene, 93*, 57 – 65.

Neilean R. L., Schaefer E., Gaff H., Fister K. R. & Lenhart S. (2010). Modeling optimal intervention strategies for cholera. *Bulletin of Mathematical Biology, 72*, 4 – 18.

Numfor, E. S. (2010). *Mathematical modeling, simulation, and time series analysis of seasonal epidemics.* M.Sc. Thesis, East Tennessee State University.

Nwagor, P. & Ekaka-a, E. N. (2017). Sensitivity of a mathematical model of HIV infection with a fractional order characterization. *International Journal of Pure and Applied Sciences, Cambridge Research and Publications, 10*(1), 86 – 92.

Ochoche, J. M. (2013). A mathematical model for the transmission dynamics of cholera with control strategy. *International Journal of Scientific and Technology Research, 2*(11), 212 – 217.

Panja, P. (2019). Optimal control analysis of a cholera epidemic model. *Biophysical Review and Letters, 14*, 27 – 48.

Rodrigues H. S., Monteiro M. T. & Torres D. F. (2013). Sensitivity analysis in a dengue epidemiological model. http://dx.doi.org/10.1155/2013/721406

Stockholm, F. (2006). Methods for uncertainty and sensitivity analysis: Review and recommendations for implementation in ecology. *Eysicum, 5*, 23 – 29.

Tarantola, S. (2008). *Global sensitivity analysis*. John Wiley and Sons.

Umoh, E.S. (2022). Mathematical Modelling Approach for the Sensitivity and Stability Analyses of Cholera Diseases in Aquatic Environment. M.Sc. Thesis submitted to the Department of Mathematics and Statistics, Ignatius Ajuru University of Education, Port Harcourt, Rivers State.

WHO (2019). Cholera – World Health Organization. https://www.who.int/news-room/fact-sheets/detail/cholera

Wang, J. & Mondak, C. (2011). Modeling cholera dynamics with controls. *Canadian Applied Mathematics Quarterly, 19(3)*, 255 – 273.

Wang, X. & Wang, J. (2014). Analysis of cholera epidemics with bacteria growth and spatial movement. *Journal of Biological Dynamics, 2, 23 – 31.*