The Application of Differential Equations in Environmental Management and Sustenance in Nigeria

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ABSTRACT

This review on the application of differential equations in environmental management and sustenance in Nigeria aimed at identifying certain aspects of environmental related phenomenon which can be managed and sustained using differential equations. It was discovered that differential equations can be applied in the area of radioactive decay and cooling of a body. There is no doubt therefore that a thorough grounding in the theory and application of differential equations should be a part of the scientific education of every scientist, both physical and social. Sequel to the above assertions, this study conclusively submitted that differential equation is highly essential in the area of Mathematics - Analysis, Algebra and Topology, and therefore should be given due attention as its wide applications in the global system can not be overemphasized.

Keywords: *differential equations, applications, environmental management, sustenance*

INTRODUCTION

Like the late seventeenth-century work in Calculus, the earliest discoveries in differential equations were first disclosed in letters from one mathematician to another, many of which are no longer available. Perhaps since astronomy was the physical field of interest that dominated the century, most of such announcements were limited to studies in planetary motion. Today, the theory of differential equations plays a fundamental role in both pure and applied mathematics. Indeed, some of the most important ideas in Analysis, Algebra and Topology were developed in attempts to resolve particular problems involving equations of this nature. The startling growth of computer in which large systems of differential equations are solved quickly and accurately has made the formulation of scientific problems in terms of differential equations more meaningful than ever before. A differential equation is a relationship between an independent variable *X*, a dependent variable *y*, and one or more derivative of *y* with respect to *X*;

$$\text{e.g } X^2 \frac{dy}{dx} - y \sin x = 0, \quad \frac{xyd^2y}{dx^2} + \frac{ydy}{dx} + e^{3x} = 0$$

The order of a differential equation is given by the highest derivative involved in the equation; e.g.

$$X\frac{dy}{dx} - y^2 = 0 \; 1^{st} \text{ order} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\text{I})$$

$$d^2y/dx^2 + 2dy/dx + 10y = \sin 2x - 2^{nd} \text{ order} \ldots\ldots\ldots\ldots\ldots\ldots (II)$$

Because, in the 2nd equation, the highest derivative involved is

$\frac{d^2y}{dx^2}$ (O'Neil, 1991).

Formation of Differential Equations

Differential equations may be formed in practice from a consideration of the physical problems to which they refer. Mathematically, they can occur when arbitrary constants are eliminated from a given function e.g.

Consider $y = A \sin X + B \cos x$, where A and B are arbitrary constants. If

we differentiate, we have:

Which is identical for the original equation, but with the sign changed

i.e $\frac{d^2y}{dx^2} = -y$ therefore $\frac{d^2y}{dx^2} + y = 0$ (this is a differential equation of the 2nd order)

A function with an arbitrary constant gives a 1st order equation, while a function with two and three arbitrary constants give a 2nd and 3rd order equation respectively, so without working each out in detail, we cay say that:

1. $y = e^{-2x}(A + Bx)$ would give a 2nd order differential equation;
2. $y = A\left[\frac{x-1}{x+1}\right]$ would give a 1st order differential equation (Boyce, 2001).

Solution of Differential Equations

To solve a differential equation we have to find the function for which the equation is true. This means that we have to manipulate the equation so as to eliminate all the derivatives and leave a relationship between y and x, this is the 1st order differential equation. Every function which, when substituted, together with its derivative into the given differential equation, turns the equation into an identity is called "a solution of the differential equation".

The more general ordinary differential equation of the 1st order is an equation of the form: $y^1 = f(x, y)$_________$*1$

Another example of an ordinary differential equation of extensive practical application is the equation: $a(x)y'' + b(x)y' + c(x)y = f(x)$____$*2$

Where *a*, *b*, *c* and *f* are given functions of *x*. This is called a linear second - order ordinary differential equation. In equation *2 above, the right member f (x), is identically zero, then the equation is referred to as “Homogenous linear equation”, otherwise is called non-homogenous.

Differential equation can also be specified according to their degree in some circumstances; and it often take the form of a polynomial expression in the unknown function and its derivatives, the degree being the exponent of the highest-ordered derivatives. It is clear that all linear differential equations are of the 1st degree except

(Erwin, 1979).

Methods of Solving Differential Equation

1 Homogenous Equations - By Substitution Y = VX:

$\frac{dy}{dx} = \frac{x+3y}{2x}$ is an example of a homogenous differential equation

This is determined by the fact that the total degree in *x* and *y* for each of the terms involved is the same (in this case, of degree 1). The key to solving every homogenous differential equation is to substitute $y = vx$, where *v* is a function of *x*.

The equation above looks simple enough, but we find that we cannot express the RHS in the form of x-factors and y-factors . In this case, we take $y = vx$.

Differentiate wrt x (using the product rule).

$ay^2 + by + c = 0$

$\frac{dy}{dx} = v.1 + \frac{xdv}{dx} = v + \frac{xdv}{dx}$, also $\frac{x+3y}{2x} \quad \frac{x+3vx}{2}$

The equation now becomes $v + \frac{xdv}{dx} \; 1 + \frac{3v}{2} = \frac{xdv}{dx} = \frac{1+3v}{2} - v$

$1 + \frac{3v-2v}{2} = \frac{1+v}{2}$. The equation is now expressed in terms of v and x, and in the form, we find that we can solve by separating the variables:

$\int \frac{2dv}{1+v} = \int \frac{1dx}{x} \; 2 \; In \; (1+v) = In \; x + C = In\,x + In\,A$

$(1+v)^2 \; Ax$, but $y = vx$, $v = y/x$, therefore, $\left(1 + (y/x)^2\right) = A = (x+y)^2 = Ax^3$ (Erwin, 1979).

Linear Equations - Use Of Integrating Factor:

Consider the equation $\frac{dy}{dx} + 5y = e^{2x}$

This is clearly an equation of the 1st order, but different from those we have dealt with so far.

In this case, we begin by multiplying both sides by e^{5x}.

This gives $e^{5x}\, dy/dx + y\; 5e^{5x} \;\; e^{2x}.e^{5x} = e^{7x}$. We now find that the LHS is, in fact the derivatives of $y.e^{5x}$, therefore, $\frac{d}{dx}\left(ye^{5x}\right) = e^{7x}$

Integrating both sides with $x: ye^{5x} = \int e^{7x}dx = \frac{e^{7x}}{7} + c$

$Y = e^{2x} + ce^{-5x} *$ N/B: Remember to divide *c* by e^{5x}

The equation we've just solved is an example of a set of equations of the form $\frac{dy}{dx} + py = Q,$ where *P* and *Q* are functions of *x* (or constants).

This is called a "linear equation of the 1st order" and to solve any such equation, we multiply both sides by an "Integrating factor" which is always $e^{\int pdx}$. This converts the LHS into the derivative of a product (O'Neil, 1991).

In our pervious examples, $\frac{dy}{dx}\ 5y = e^{2x}, P = 5$

Therefore $\int pdx = 5x$ and the integrating factor was therefore e^{5x}.

NOTE: In determining $\int pdx$, we do not include a constant of integration. This omission is purely for convenience, for a constant of integration here would in practice give a constant factor on both sides of the equation which would subsequently cancel.

Exact Differential Equations

The differential equation, $M(x, y)dx + N(x, y)dy = 0$ ________ $*1$ is called an "exact" if and only if *M(x,y)* and *N(x,y)* are continuous differential functions for which the following condition is fulfilled:

and are continuous in some region

(Boyce, 2001).

In integrating exact differential equations, we shall prove that if the left hand side of equation *1 is an exact differential, then condition * is fulfilled.

Let us first assume that the left hand side of * is an exact differential of some function

$du\ \frac{du}{dx} + \frac{du}{dy}dy$ implies $M = \frac{du}{dx},$ and by differentiating the first relation with respect to *y* and the second with respect to *x*, we obtain

$dM = \frac{d^2u}{dxdy} \quad \frac{dN}{dx} = \frac{d^2u}{dydx},$ therefore condition (*) is a necessary condition for the left hand side of *l to be an exact differential (O'Neil, 1991).

Let's see from the relation $\frac{du}{dx} = M(x, y)$ we find

$U = \int_{xo}^{x} M(x,y)dx + Q(y \_\_\_\_\_\_\_\_ (*)_2)$ where x_0 is the abscissa of any point in the domain of the existence of the equation.

$$\frac{du}{dx} = \int_{xo} \frac{dM}{dy} dx + \Phi^1(y) = N(x,y) \text{ but } \frac{dM}{dy} = \text{hence } \int_{ox}^{x} \frac{dN}{dx} dx$$

$$+\Phi^1(y) = \Phi\sqrt{(x,y)} \text{ i.e } N(x,y)]^{x}_{xo} + \Phi^1(y)$$

$$= N(x,y) \text{ or } N(x,y) - N(x_0 y) + \Phi^1(y)$$

$$= N(x,y)$$

Hence,

$$\Phi^1(y)\ N(x_0, y) \text{ or } \Phi(y) = \int N(x_o, y)dy + B$$

Applications of Differential Equations in Environmental Management and Sustenance

Differential equations provide alternative descriptions for the real world and constitute a formidable tool convenient for modern computers. To better appreciation the role of differential equations, our attention will be restricted only across the spectrum of science.

Radioactive Decay

It is established experimentally that the rate of radioactive decay is proportional to the remaining mass of the substance. If *N* equals the mass of radioactive substance present at any time *t*, the rate of change of *N* is described by the equation:

$\frac{dN}{dt} = -kN$, were *k* is proportionality constant.

The negative sign shows a decrease in the amount of the substance as *t* grows, and we write $\frac{dN}{dt} = -kdt,\ InN = -kt + InC$

$$\text{or } N = Ce^{-kt}$$

The value of the constant k can be determined experimentally by measuring the amount of the remaining substance at a time moment *t*. The rate of decay is usually characterized by the so called half-life period, *T* (Erwin, 1979 and James, 1999).

$$\frac{N_0}{2} = N_0 e^{-kt} \text{ and } e^{-kt} = 2$$

Radiocarbon dating is a technique used by archeologists to determine the ages of fossils uncovered during excavation (O'Neil, 1991).

Cooling of a Body

According to the law established by Newton, the temperature of a cooling body drops at a time-rate proportion to the difference between the temperature of the body and the temperature of the surrounding medium, i.e $\frac{dT}{dt} = -K(T - Ts)$, where *T* is the temperature of body (James, 1999).

Ts = temperature of surrounding

By separating the variables, we've: $\frac{dT}{T - Ts} = -kdt$

In $(T - Ts) = -Kt + LnC, T = Ts + Ce^{-kt}$

CONCLUDING REMARKS

The aim of this study was to identify certain aspects of environmental related phenomenon which can be managed and sustained using differential equations. The study highlighted that differential equations can be applied in the area of radioactive decay and cooling of a body. Hence, for the environmental management and sustenance agenda to work well in the area of Mathematics, the core area of Mathematics (pure and applied), the differential equations with its wide applications should be given priority if the entire Science is to be enhanced experimentally through manpower training and adequate workshops organized by the abroad-trained Mathematicians and Engineers. Differential equation is highly essential in the area of Mathematics - Analysis, Algebra and Topology, and therefore should be given due attention as its wide applications in the global system can not be overemphasized.

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