USING MAPLE SOFTWARE TO ACHIEVE POLYNOMIAL OF DEGREE FIVE (5) FROM NUMEROV FORMULA

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ABSTRACT

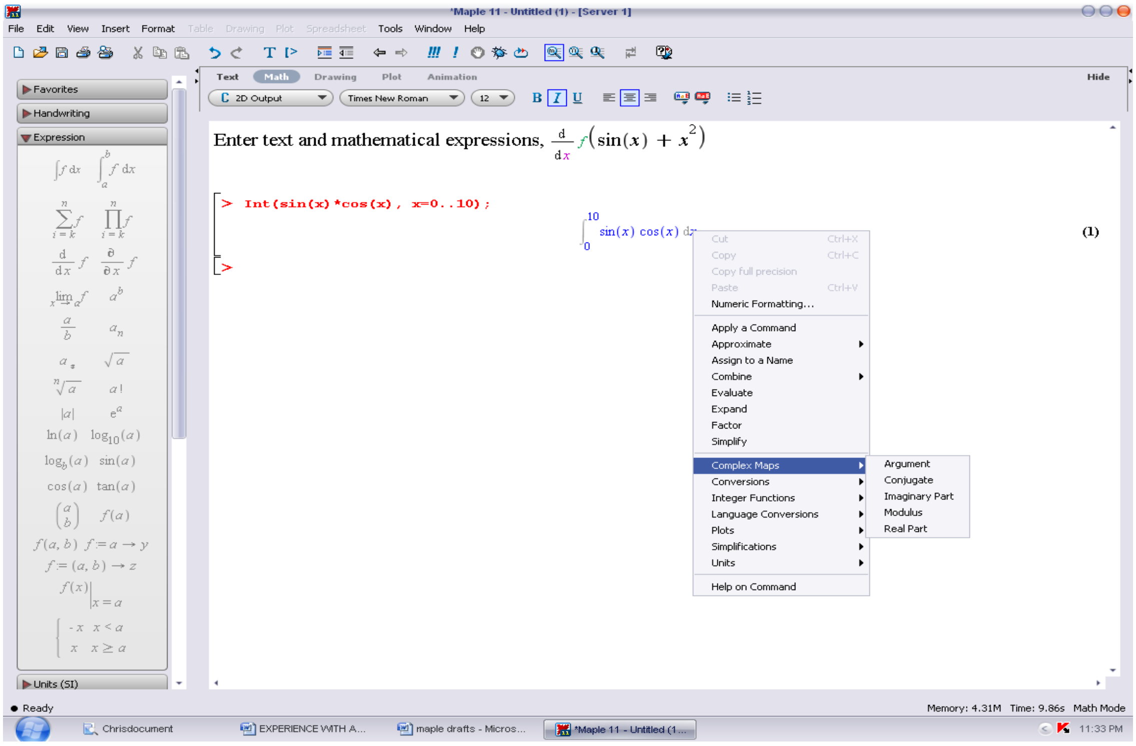
This work was aimed at exploring the use of Maple Software, an ideal mathematical tool, particularly, in the derivation of a Numerov Method for the solution of a Second Order Ordinary Differential Equation (ODE) of the type $y'' = f(x,y), \quad y(x_0) = y_{0,} \quad y'(x_0) = z_0$ *by a polynomial of degree five on the mesh points. The result obtained agreed with the derivation of a Continuous Numerov Formula, Yahaya (2004). It was possible to use the Maple software for the symbolic computation of the matrix AX = B from the polynomial of degree five to achieve result. It also aimed at achieving a polynomial of degree five with the same initial conditions satisfied by the polynomial of degree four at* $x_i, \; x_{i+1}, \; x_{i+2}$. *It was encouraging to compare the volume of work achieved in less time using the software than by manual computation.(paper and pen based). The result is optimal for the derivation of approximation on the three grid points . Consequently, it is justifiable to encourage the use of Maple Software in higher institutions of learning.*

INTRODUCTION

Maple is the ideal mathematical software for technicians, professionals, researchers, educators and students. With over 3500 routines, Maple covers a broad spectrum of concepts from introductory calculus to Fast Fourier Transforms. Also known as MapleSoft, it is a general - purpose commercial computer algebraic system. It was developed in 1980 by the Symbolic Computation Group at the University of Waterloo, Ontario, Canada. The main aim for this research is to achieve a polynomial of degree 5 with the same initial conditions satisfied by the polynomial of degree 4 at $x_i, \; x_{i+1}, \; x_{i+2}$. This is with the hope that at off grid point, the polynomial of degree 5 should lead to a more accurate approximation.

Worksheet Interface

When entering information at the Maple input prompt (>), entries display in red font. Maple output displays below the Maple input in blue font. Worksheets can contain sections, text regions, hyperlinks, symbolic spreadsheets, sketch regions, and more. Context-sensitive menus and palettes provide alternative methods of entering input. Users can enter mathematics in traditional mathematical notation. Custom users' interfaces can also be created. There is support for numeric and symbolic computation, as well as visualization.



Examples of Maple Codes

Assigning Values to Variables: Assigning values to variable names gives you more control when working through a problem. Values can be numeric, symbolic, or a combination of the two. To assign a value to a variable, use the assignment operator (**:=**). Some names in Maple are protected; therefore, you receive an error message if you attempt to assign a value to a protected name.

Assign a value to x

> x := 5;

Assign an expression to **expr**.

> expr: = y^2 + 7*y + 12;

Solving Equations: Maple contains routines to solve different types of equations, including systems of equations and differential equations.

To solve an equation symbolically, use the **solve** command.

> solve(x^4-5*x^2+6*x = 2, {x}):

To solve a system of equations containing several variables, use the **solve** command.

> solve({x-3*y = 5, 5*x-3*y = 7}, {x, y});

To solve an equation numerically, use the **fsolve** command.

> fsolve(tan(sin(x)) = 1, {x});

Calculus: Maple provides many powerful tools for solving problems in single and multi-variable calculus. You can use Maple to solve problems numerically and symbolically in the areas of differentiation, integration, limits, and many more.

Differentiate an expression.

> diff(sin(x)*cos(x), x);

With Maple, you can perform definite and indefinite integration.

> int(cos(x), x);

To calculate a definite integral, specify an interval for x as indicated in the following example.

> Int(ln(x), x=0..10);

The **Int** command represents an expression, but does not evaluate it. To calculate a numeric approximation of this expression, use the **value** command.

> value(%);

Linear Algebra: Maple has a robust and extensive set of commands for working with Vectors and Matrices.

Define a Matrix and a Vector.

> M := Matrix([[2,3,6], [6,6,3], [8,9,1]]);

$$M := \begin{bmatrix} 2 & 3 & 6 \\ 6 & 6 & 3 \\ 8 & 9 & 1 \end{bmatrix}$$

> V := Vector([a, b,c]);

$$V := \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

“ **.**” for matrix multiplication and ” *****” for scalar multiplication.

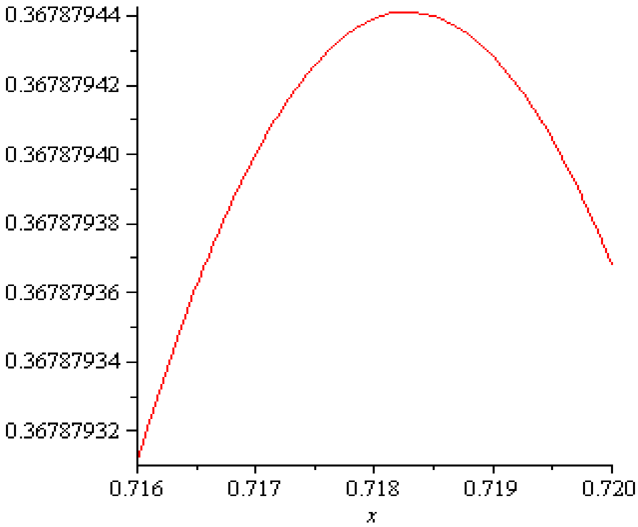
> 3*M.V;

$$\begin{bmatrix} 6\,a + 9\,b + 18\,c \\ 18\,a + 18\,b + 9\,c \\ 24\,a + 27\,b + 3\,c \end{bmatrix}$$

Plotting of function of single variable

Plot $\frac{\ln(x+2)}{x+2}$ with x ranging from 0.716 to 0.720

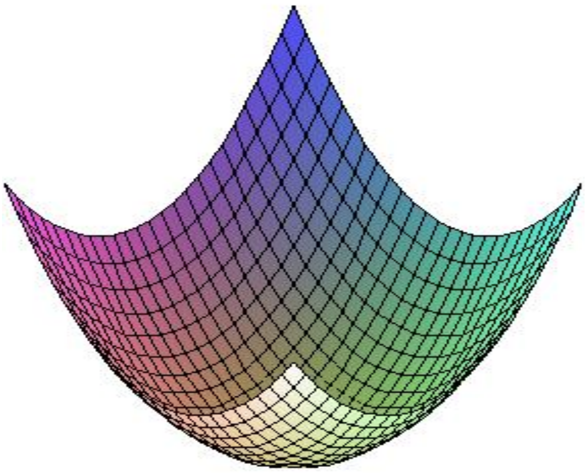
>plot(ln(x+2)/(x+2), x = .716 .. .72);



Plotting of function of two variable

Plot $x^2 + y^2$ with x ranging from -1 to 1

>plot3d(x^2+y^2, x=-1..1,y=-1..1);



Continuous Numerov Formular

The derivation of Linear Multistep Methods (LMM) through interpolation and collocation is a well-known process. Collocation has been describe as probably now the most important numerical procedure for obtaining continuous methods for ODE's as evident in Onumanyi (2008), Onumanyi, Fatokun and Adejo (2008), Lambert (1973, 1991) and Awoyemi (2002); Awoyemi (2003), Awoyemi and Kayode (2002); Awoyemi and Kayode (2003); Bun and Vasil'Yev (1992); Faires and Barden (1993), Fatokun (2005), Fatokun (2007), Fatokun (2007), Jacques and Judd (1987), Kimbir (2008).

Yahaya (2004) obtained a Continuous Numerov Formula $U(x)$;

$$U(x) = \frac{[-(x - x_{n+1})]}{h} y_n + \frac{[h + (x - x_{n+1})]}{h} y_{n+1} + \frac{[(x - x_{n+1})^4 - 2h(x - x_{n+1})^3 + 3h^3(x - x_{n+1})]}{24h^2} f_n + \frac{[-(x - x_{n+1})^4 + 6h^2(x - x_{n+1})^2 + 5h^3(x - x_{n+1})]}{12h^2} f_{n+1} + \frac{[(x - x_{n+1})^4 + 2h(x - x_{n+1})^3 - h^3(x - x_{n+1})]}{24h^2} f_{n+2} \qquad (1.1)$$

from a polynomial of degree 4. Evaluating (1.1) at $x = x_{n+2}$ we have;

$$-y_n + 2y_{n+1} + \frac{1}{12}h^2 f_n + \frac{5}{6}h^2 f_{n+1} + \frac{1}{12}h^2 f_{n+2} \qquad \ldots \qquad (1.2)$$

Considering the first derivative function derived from the continuous method (1.1), we have

$$\frac{dU}{dx}(x) = Z(x), \; \frac{dU}{dx}(a) = z_0 \qquad \ldots \qquad (1.3)$$

The simultaneous application of (1.1) and (1.2) leads to the Finite Difference Methods (FDMs) of the form

$$y_{n+2} - 2y_{n+1} + y_n = \frac{h^2}{12}\left(f_{n+2} + 10f_{n+1} + f_n\right) \qquad \ldots \qquad (1.4)$$

$$Order\ 4, \qquad C_6 = -\frac{1}{240}$$

Combining (1.3) when $n = 0$ together with $\frac{dU}{dx}(a) = z_0$ in (1.2) explicitly, we have

$$y_2 - 2y_1 + y_0 = \frac{h^2}{12}\left(f_2 + 10f_1 + f_0\right) \qquad \ldots \qquad (1.5)$$

$$y_1 = y_0 + h_0 y'_0 + \frac{h^2}{24}\left(7f_0 + 6f_1 - f_2\right) \qquad \ldots \qquad (1.6)$$

GETTING STARTED WITH THE NUMEROV FORMULA BY A POLYNOMIAL OF DEGREE 5 ON THE MESH POINTS

Let the approximate solution to problem (1.1) be a partial sum of a power series of the form:

$$y(x) = \sum_{j=0}^{5} a_j x^j \qquad \dots \qquad (2.1)$$

taking the second derivative of (1.1) and using this in equation (2.1) yields;

$$\sum_{j=2}^{5} j(j-1)\, a_j x^{j-2} = f(x, y, y') \qquad \dots \qquad (2.2)$$

Collocation points are taken in equation (1.2) at all grid points, $x = x_{n+i},\ i = 0(1)2$. Equation (1.1) is interpolated at all grid points, $x = x_{n+i},\ i = 0(1)2,\ x \epsilon \mathbb{R}$, where $\mathbb{R}$ is the set of real numbers. The system obtained from the collocation and interpolation above is represented by the matrix equation:

$$AX = B \qquad \dots \qquad (2.3)$$

Where A, a 6×6, X and B are respectively given by;

$$A_{6\times 6} = \begin{bmatrix} 0 & 0 & 2 & 6x_n & 12x_n^2 & 20x_n^3 \\ 0 & 0 & 2 & 6x_{n+1} & 12x_{n+1}^2 & 20x_{n+1}^3 \\ 0 & 0 & 2 & 6x_{n+2} & 12x_{n+2}^2 & 20x_{n+2}^3 \\ 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 \\ 1 & x_{n+2} & x_{n+2}^2 & x_{n+2}^3 & x_{n+2}^4 & x_{n+2}^5 \end{bmatrix}$$

$$B = [y_{n,}\ y_{n+1}, -y_n + 2y_{n+1} + \frac{1}{12}h^2 f_n + \frac{5}{6}h^2 f_{n+1} + \frac{1}{12}h^2 f_{n+2}, f_n, f_{n+1}, f_{n+2}]^T$$

$$X = [a_{0,}\ a_1, a_2, a_3, a_4, a_5]^T$$

The term $-y_n + 2y_{n+1} + \frac{1}{12}h^2 f_n + \frac{5}{6}h^2 f_{n+1} + \frac{1}{12}h^2 f_{n+2}$ in vector B is a known scheme of $Order\ 4,\ C6 = -\frac{1}{240}$ gotten from the evaluation of the continuous scheme (1.1) from a polynomial of degree 4 at $x = x_{n+2}$. This is with the hope that a better accuracy for the proposed scheme be achieved.

Using the Row Echelon Form method in the Maple Software to solve the matrix equation (2.3), the values $a_j's,\quad j = 0,1,\dots,5$ are obtained as shown in the appendix. These values of $a_j's$ are substituted into equation (2.1) and interpolating at the off grid point $x = x_{n+3/2}$ we have the continuous scheme as shown equation (2.4).

$$y_{n+3/2} = \frac{3}{2}y_{n+1} - \frac{1}{2}y_n + \frac{h^2}{128}[-f_{n+2} + 42f_{n+1} + 7f_n] \qquad \dots \qquad (2.4)$$

$$Order\ 3,\ C_5 = \frac{5}{768} \approx \frac{1}{153.6}$$

Equating equation (2.4) and

$$y_{r+1} - y_r - h_r y'_r = \frac{h_r^2}{360}\left(33 f_{r+2} - 128 f_{r+\frac{3}{2}} + 186 f_{r+1} + 89 f_r\right) \quad \ldots \quad (2.5)$$

(Onumanyi et al, 2002, hybrid method of 2008) and simplifying, we have,

$$y_{n+1} = y_n + h_n y'_n + \frac{h^2}{24}\left(7f_n + 6f_n - f_n\right) \quad \ldots \quad (2.6)$$

at $n = 0$, we have,

$$y_1 = y_0 + h_0 y'_0 + \frac{h^2}{24}\left(7f_0 + 6f_1 - f_2\right) \quad \ldots \quad (2.7)$$

$$order\ 3, \qquad C_4 = 0\,, \qquad C_5 = \frac{1}{45}$$

RESULT FROM THE POLYNOMIAL OF ORDER 5

The results from this method have agreed with the earlier Quiroz (Gonzalez and Thompson, 1997), the Taylor series method and by derivation of Continuous Numerov Formula (Yahaya, 2004). Thus confirming by the method, equation 2.7 is optimal for the derivation of approximation on the three grid points $x_i,\ x_{i+1},\ x_{i+2}$ for a polynomial of degree 5. The findings of this research have added to a growing body of literature on the subject of Numerov Methods for the direct solution of Second Order initial value problems of Ordinary Differential Equations. The use of the Maple software has been very useful in the research work as it speeds up the volume of work involved in the computation of the problem (2.3), $AX = B$.

CONCLUDING REMARK

The aim of this study was to achieve a polynomial of degree 5 with the same initial conditions satisfied by the polynomial of degree 4 at $x_i,\ x_{i+1},\ x_{i+2}$. Our experience with Maple Software is worthwhile and thus the use of Mathematical Software like Maple Soft should be encouraged in institution to aid learning. From the method developed (through a polynomial of order five) a number of possible future studies using the same experimental set up are apparent and thus should not be overlooked. This information can be used to develop targeted interventions aimed at producing greater approximations in future research works.

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